EXERGY BASED EFFICIENCY ASSESSMENT OF FANS VS ISENTROPIC EFFICIENCY

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SUMMARY

The efficiency definition allows the comparison of two machines with each other. In general, the efficiency is defined as the ratio of usable power to the required power. This raises the question: what is the usable power? Most engineers discuss efficiency on grounds of the energy balance, i.e. the first law of thermodynamics. In this paper, we derive the exegetic efficiency taking the second law of thermodynamics into account, too. On this basis, a comparison between isentropic and exergetic efficiency is given. A high-pressure radial fan is used as an example and the differences are discussed. Therefore, measurements at a non-adiabatic fan is evaluated and the role of the heat flux to the environment is discussed. For a standard such as ISO 5801 "Fans - Performance testing using standardized airways", efficiency must not only be physically correct. It must also be simple and practical. Against this background, the outlook of this paper discusses when and which efficiency definition is appropriate and best suited for a standard.

INTRODUCTION

Fans and compressors are working machines that are characterized by their wide operating range and their application in every industrial sector. Their fundamental function is to deliver a volume flow of a fluid at a given pressure difference, which makes them essential for all kinds of industrial processes, from simple compressed air supply to highly complex chemical reactions. Most diverse application-related requirements and gases lead to a variety of machine designs. In this regard, physical working principles give a reasonable structure for fans and compressors. For today's technical applications, the two most relevant principles are the hydrodynamic and hydrostatic principle. The associated machine categories are turbomachines and positive displacement machines. In this paper we focus on the application for fans.

Besides the importance of fans and compressors for the implementation of industrial processes, their use causes costs. These costs mainly depend on the energy consumption considering their life cycle costs. This is why investments in improving energy efficiency of working machines like fans or compressors usually show a high cost effectiveness with a low financial risk. However, energy consumption and their related costs not only have an impact on their users but also have a relevance to society. Detailed studies under the European Commission's Ecodesign Directive considered fans and compressors to be relevant electric energy consumers [1]. In this context, a recent study by the German Ministry of Economics and Energy shows that pumps and compressors account for 16 % of electrical energy consumption in German industry [2]. Since electricity production worldwide is still heavily dependent on fossil fuels, fans and compressors consequently contribute significantly to CO_2 emissions and thus to climate change and the associated future costs to society.

Consequently, from the perspective of both industry and society an energy efficiency assessment of fans and compressors is highly relevant. The efficiency represents the dimensionless measure of the dissipative power loss of a machine. At this point, it is necessary to differentiate between machine, module and system. The task of a turbomachine or positive displacement machine is to convert mechanical power into fluid power. The system boundaries are the shaft of the machine as well as the inlet and outlet of the machine. A module on the other hand consists of more than one machine or component and is also called “extended product”. In the context of fluid working machines, a module usually includes a frequency converter, an electric motor and the pump or the compressor. Their energetic assessment is based on a load scenario including part load and their efficiency characteristic. Indeed, part load is much more common than operation in best point. Finally, technical systems usually consist of multiple machines and components realizing technical processes. In the case of fluid power systems there exist absolute measures based on physical axioms. On the one hand, Betz law gives an energy harvesting factor for wind turbines that is defined by the ratio of mechanical power to the available power with an upper limit of 16/27 [3]. On the other hand, Pelz gives an upper limit for hydropower in an open-channel flow of ½ [4]. The most famous absolute measure is the Carnot efficiency, which defines the maximum efficiency of an ideal heat engine or Carnot cycle [5]. In this case, the exergy becomes the relevant quantity which measures the working capacity of a fluid relative to its environment. Exergy was first introduced by Fritsche, Hehnemann and Rant in 1956 [6]. While exergy analyses are state of the art in evaluating thermal power or working systems [7], exergy based efficiency studies for single fluid working machines are rare [8]. Nevertheless, we expect relevant insights for diabatic machines where the heat flow is not zero.

Against this background, we make two conclusions. Firstly, the efficiency of fluid working machines is of fundamental importance for their own assessment and for the assessment of both modules and working systems. At the same time, despite of countless scientific studies on the efficiency of fluid working machines, the definition of efficiency, the measuring methods and the application in standards and directives are ongoing issues, e.g. the Revision of ISO 5801:2018 for fans. Secondly, an exergy-based assessment for fans and compressors is necessary and leads to the most general definition of the efficiency. Therefore, this paper begins with the derivation of the exegetic efficiency based on the first and second law of thermodynamics. Following this general form, we consider the application on a high-pressure centrifugal fan. Furthermore, the isentropic efficiency definitions are compared to the exegetic efficiency. Subsequently, we discuss reference applications and compare different efficiency definitions based on measurement data. The paper closes with the conclusion and outlook.

THE EFFICIENCY IN THE LIGHT OF THE FIRST AND SECOND LAW OF THERMODYNAMICS

Most engineers discuss efficiency only on grounds of the energy balance, i.e. the first law of thermodynamics. In this section we derive the exegetic efficiency also taking into account the second law of thermodynamics. For this most general definition of the efficiency, we will see, that both axioms are indeed needed.

The first law of thermodynamics for a machine operating a steady state reads

$$\dot{m}(h_{t2} - h_{t1}) = P_S + \dot{Q}. \tag{1}$$

This law is a conservation of energy, distinguishing heat flow $\dot{Q}$ and work P_S, and relating them to the internal energy, comprising the mass flow rate $\dot{m}$ and the difference in enthalpy $h_{t2} - h_{t1}$.

The second law of thermodynamics (entropy theorem) states

$$\dot{m}(s_2 - s_1) = \Delta\dot{S}_{irr} + \frac{\dot{Q}}{T}, \tag{2}$$

with the difference in mass specific entropy $s_2 - s_1$ on the right hand. On the left hand, the source term $\Delta\dot{S}_{irr}$ is the work dissipated within the system: work which does not reach the outside but increases the internal energy as a result of friction, throttling or shock processes. The second term $\dot{Q}/T$ considers an increase or decrease in entropy as a result of heat flow across the system boundary.

Zoran Rant [6] introduced the concept of exergy in 1956. The exergy describes the actual working ability of a fluid. The temperature T in the 2nd law of thermodynamics is set to the ambient temperature T_0. By multiplying the 2nd law by T_0 and then subtracting the 1st and 2nd laws, the following expression is obtained:

$$\dot{m}(h_{t2} - h_{t1}) - T_0\dot{m}(s_2 - s_1) = P_s - T_0\Delta\dot{S}_{irr} \tag{3}$$

Exergy is the working part of energy and the result of subtracting energy and anergy:

exergie := energy - anergy

$$ex_2 - ex_1 := h_{t2} - h_{t1} - T_0(s_2 - s_1) \tag{4}$$

The interpretation of the right side of equation 3 shows that the exergy is the fraction of the shaft power that is transmitted to the fluid and does not dissipate.

The function of a fan or compressor is to increase the exergy of the delivered gas. The exergy measures the working capacity of the gas relative to its environment. The mass-specific exergy of a fluid particle before entering the turbomachine (see Figure 1) is defined as

$$ex_1 := h_{t1} - h_0 - T_0(s_1 - s_0)\,. \tag{5}$$

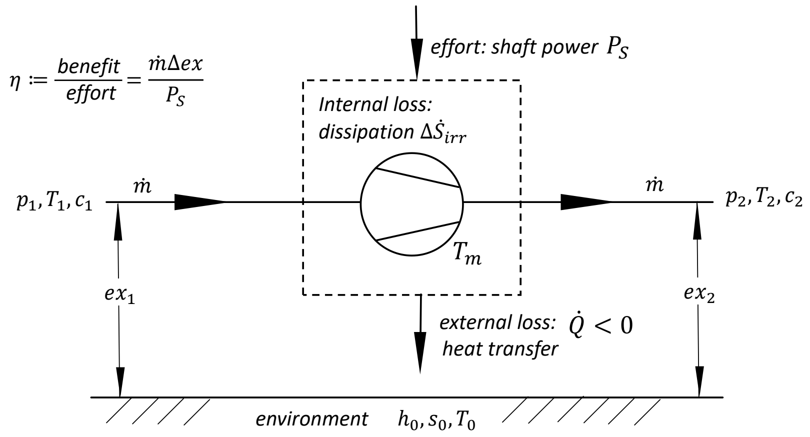

Figure 1: System boundaries of a turbomachine with inlet, outlet conditions and exchange with the environment.

In passing the fan, the exergy of the same fluid particles is increased to be

$$ex_2 := h_{t2} - h_0 - T_0(s_2 - s_0)\,. \tag{6}$$

Hence, the *benefit* of the machine is given by the change in flux of exergy

$$\text{benefit} = \dot{m}\,(ex_2 - ex_1) = \dot{m}\,\Delta ex. \tag{7}$$

The benefit is the product of a "flux" $\dot{m}$ and a difference in potential Δex. The *effort* to reach this benefit is the shaft power P_S. It is the mechanical work performed on the fluid per unit time.

Hence, the most natural definition of a machine efficiency, fan or compressor, is

$$\eta_{\text{ex}} := \frac{\dot{m}\,\Delta ex}{P_S}. \tag{8}$$

For the consideration of a heat flow $\dot{Q}$, we go back to the 1st and 2nd law of thermodynamics (eq. 1 and 2). We adhere to the usual norm, were $\dot{Q}$ is positive as long the heat flux is into the machine. But for turbomachines the heat flux is in most cases from the machine to the environment, i.e. $\dot{Q} < 0$. Hence, the temperature T in the second law of thermodynamics is the temperature of the source of the heat conduction. For $\dot{Q} < 0$ the pure temperature is the average machine temperature being larger than the ambient temperature $T = T_m > T_0$.

Replacing the total enthalpy in the 1st law of thermodynamics by exergy and setting $T = T_m$ the two axioms are written as

$$\dot{m}(ex_2 - ex_1) = P_S + \dot{Q} - T_0\dot{m}(s_2 - s_1), \tag{9}$$

$$\dot{m}(s_2 - s_1) = \Delta\dot{S}_{irr} + \frac{\dot{Q}}{T_m}. \tag{10}$$

We are now in the position that the entropy difference $\Delta s = s_2 - s_1$ can be eliminated from the two axioms:

$$\dot{m}(ex_2 - ex_1) = P_S + \dot{Q}\left(1 - \frac{T_0}{T_m}\right) - T_0\,\Delta\dot{S}_{irr}. \tag{11}$$

This equation, based on axiomatic grounds, is most instructive when discussing efficiency. The first and second law of thermodynamics gives now the clear interpretation of the exergetic efficiency:

$$\eta_{\mathrm{ex}} := \frac{\dot{m}\Delta ex}{P_S} = 1 + \frac{\dot{Q}}{P_S}\left(1 - \frac{T_0}{T_m}\right) - \frac{T_0\,\Delta\dot{S}_{irr}}{P_S}. \tag{12}$$

Now the internal and external loss mentioned above are taking shape as follows:

$$\text{inner loss} = \text{dissipation} = \varepsilon_i := \frac{T_0\,\Delta\dot{S}_{irr}}{P_S}, \tag{13}$$

$$\text{outer loss} = \text{heat transfer} = \varepsilon_o \coloneqq -\frac{\dot{Q}}{P_S}\left(1 - \frac{T_0}{T_m}\right). \tag{14}$$

It is remarkable that it is not the entire heat flow that leads to losses, but only the share

$$\eta_C := 1 - \frac{T_0}{T_m}. \tag{15}$$

This dimensionless ratio is known as the Carnotian efficiency. In conclusion, it is well known, that the fluid friction results in a dissipation rate $\Delta\dot{S}_{irr}$ reducing the efficiency to be smaller than one. It is not well known, that heat conduction $\dot{Q}$ reduces the efficiency also but only the “Carnot”-part η_C of that heat flux. With the abbreviations $q \coloneqq \dot{Q}/\dot{m}$, $w \coloneqq P_S/\dot{m}$, and the definitions (13), (14) the important equation (12) may be written in the equivalent form

$$\eta_{\mathrm{ex}} := \frac{\Delta ex}{w} = 1 - \varepsilon_o - \varepsilon_i\,. \tag{16}$$

Models for heat transfer and dissipation are necessary for the calculation. Using the equation for heat transfer $\dot{Q} = \alpha A(T_m - T_0)$ for the outer loss, the loss can be represented as follows

$$\varepsilon_o = -\frac{\alpha A T_m}{P_s}\left(1 - \frac{T_0}{T_m}\right)^2, \tag{17}$$

with the heat transfer coefficient α and the surface of the machine A. The dissipative losses are machine dependent. This will not be discussed in detail in this paper, reference is made to further literature: Pelz et al [9] give a detailed description of the losses for fans and Schobeiri [10] for compressors. In variant (i), a separate consideration of heat transport losses and dissipative losses can be made, but this method is rather unsuitable for practical application due to the effort involved.

For a caloric and thermal ideal gas with specific heat c_p, c_v, $\gamma \coloneqq c_p/c_v$ and $R = c_\mathrm{p} - c_\mathrm{v}$, the entropy is measured by measuring temperature T and pressure p [11].

$$s - s_0 = c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\ . \tag{18}$$

The absolute enthalpy is given by

$$h_t - h_0 = c_p(T_t - T_0). \tag{19}$$

Hence Δex is given by

$$\frac{\Delta ex}{c_p T_0} = \frac{T_{t2} - T_{t1}}{T_0} - \ln\left(\frac{T_2}{T_1}\right) + \frac{\gamma - 1}{\gamma}\ln\left(\frac{p_2}{p_1}\right). \tag{20}$$

With the specific work $w \coloneqq P_s/\dot{m}$ the exergetic efficiency is given by

$$\eta_{\mathrm{ex}} = \frac{c_p(T_{t2} - T_{t1})}{w} - \frac{c_p T_0}{w}\left[\ln\left(\frac{T_2}{T_1}\right) - \ln\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]. \tag{21}$$

For an ideal gas we hence have

$$\eta_{\mathrm{ex}} = 1 + \frac{q}{w}\eta_c - \varepsilon_i = \frac{\Delta h_t}{w} - \frac{c_p T_0}{w}\left[\ln\left(\frac{T_2}{T_1}\right) - \ln\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]. \tag{22}$$

Equation 22 shows two ways to calculate the exergetic efficiency:

(i) calculation by specifying the losses and

(ii) determination on calculating the generated entropy.

Variant (ii) is more suitable for the calculation of the exergetic efficiency, since this can be determined solely by the inlet and outlet variables and the ambient conditions.

In comparison to the isentropic efficiency, (eq. 23) based on isentropic compression,

$$\eta_s = \frac{\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + \frac{1}{2}\left(\frac{\dot{m}}{\rho_2 A_2}\right)^2 - \frac{1}{2}\left(\frac{\dot{m}}{\rho_1 A_1}\right)^2}{w}. \tag{23}$$

it becomes clear that the exergetic efficiency considers not only pure fluid power, but also the relation to the environment. The application to a fan shall highlight the distinctions.

APPLICATION WITH A HIGH-PRESSURE CENTRIFUGAL FAN

For the application of the exergetic efficiency, a high-pressure radial ventilator is chosen, as the differences are estimated to be more significant due to the higher temperature differences. The fan with impeller diameter $D_r = 1.33\ \mathrm{m}$ is run with the rotational speed $n = 2300\ \mathrm{U/min}$. The pressure build-up achievable with these conditions is a maximum of $\Delta p_t = 19000\ \mathrm{Pa}$ and is thus still within the scope of the ISO 5801:2018 standard.

The setup of the centrifugal fan test rigs is shown in

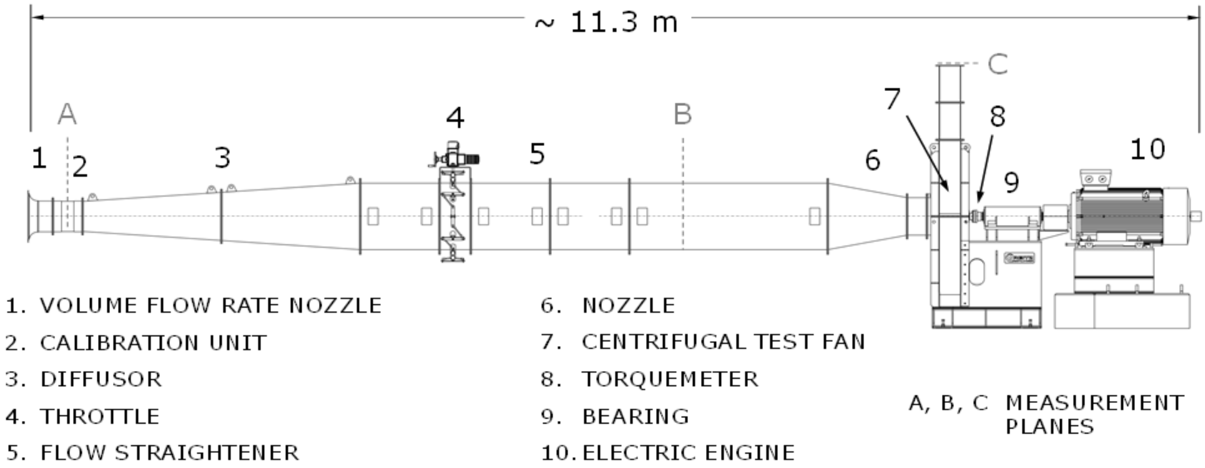

Figure 2 for the medium scaled test rig. The flow goes from left to right and passes firstly the volume flow rate nozzle (1, A). To improve the measurement accuracy the volume flow nozzle has been calibrated with a total pressure comb (2). After the throttle (4), the flow straightener (5) is placed to lower the turbulence and inhomogeneities. The fan inlet conditions (total temperature and static pressure) are measured in the measuring chamber (6) at position (B). The test fan (8) is connected with a torque meter (9) to the engine (11). The torque meter is located between impeller and bearings. The fan blows out freely into the environment. At point (C), the static pressure can thus be assumed to be equal to the ambient pressure. Only the temperature is measured at the outlet. The ambient conditions are recorded in the experimental hall by a separate device.

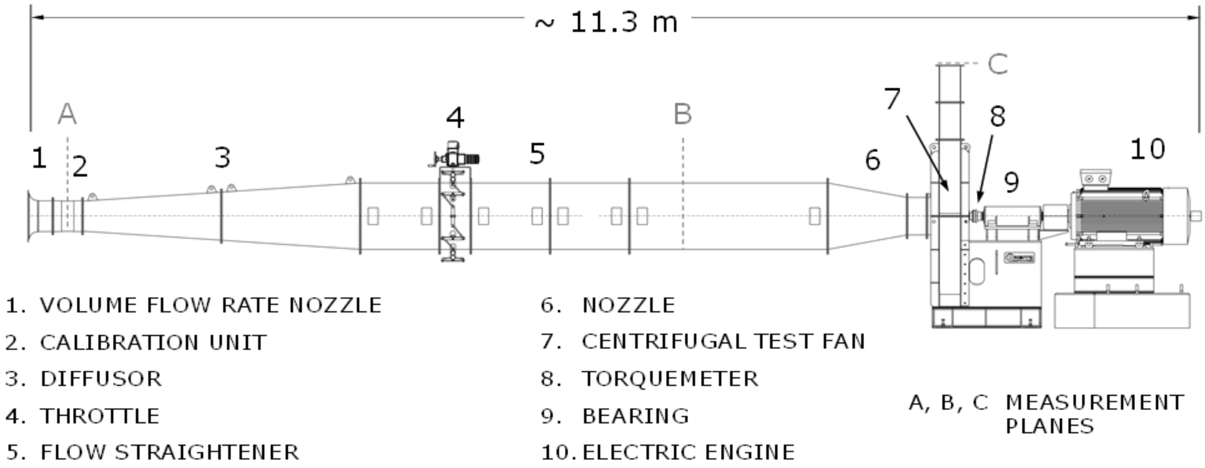

Figure 2: Pipe test rig according to ISO 5801:2018 with high-pressure centrifugal fan.

Thermal images of the spiral housing were taken to quantify the machine temperature T_m. An example is given in Figure 3, which shows the operating point of the maximum partial load. The determination of the heat flow requires the determination of the heat transfer coefficient α of the machine to the environment. The determination was complicated by the free-blowing fan. At maximum overload, mass flows of $\dot{m} = 10\,\mathrm{kg/s}$ are conveyed, which corresponds to discharge velocities of $c_2 = 144\ \mathrm{m/s}$. The exit flow leads to a circulation of the air in the area of the fan. The heat transfer coefficient is dependent on the flow around the machine and thus not constant in our experiments. By calculating the heat transfer coefficient backwards, based on the calculation of the heat flow by equation (1), the coefficients could be compared with literature values and thus checked for plausibility.

$$\dot{Q} = \dot{m}\,(h_{t2} - h_{t1}) - P_S. \tag{24}$$

$$\alpha = \frac{\dot{Q}}{A(T_\mathrm{m} - T_0)} = \frac{\dot{m}(h_{t2} - h_{t1}) - P_S}{A(T_\mathrm{m} - T_0)}. \tag{25}$$

The evaluation yielded heat transfer coefficients of $\alpha = 4.26\ \mathrm{W/m^2K}$ for stagnant air in the partial-load and $85.52\ \mathrm{W/m^2K}$ for the full-load, which corresponds to moderately to briskly moving air perpendicular to a metal wall according to the literature.

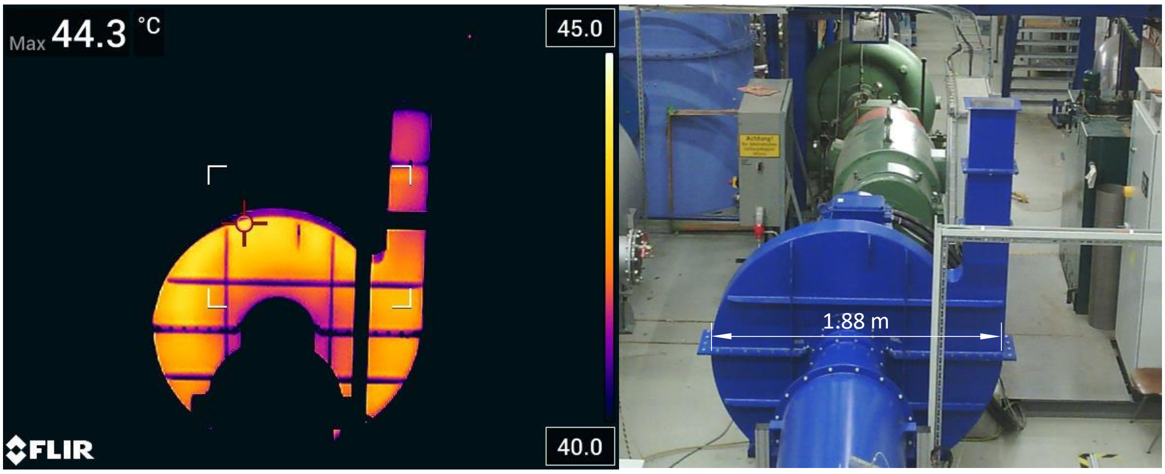

Figure 3 Thermal images of the spiral housing. FST Lab., TU Darmstadt

COMPARISON OF EFFICIENCY REPRESENTATIONS

Figure 4 shows the comparison of the exergetic efficiency to the isentropic efficiency. For the isentropic efficiency, the power of the isentropic compression was set in relation to the impeller power, while for the exergetic efficiency the calculation given in equation (22) was used. The differences in the over-load operation between the two efficiencies are negligible. The efficiency difference increases towards the part-load operation with a maximum efficiency difference of 1.8 percentage points. Remarkable is the good correlation despite different calculation rules.

Where do the differences in efficiency come from? In the case of isentropic compression, a reversible change of state is assumed, which determines the integration path of the change of state. Since entropy is a state variable, the entropy difference does not depend on the choice of the integration path [12]. Accordingly, the exergetic efficiency is also independent of the choice of an integration path.

The exergetic efficiency includes not only the compression power but also the reversible thermal power, related to the environment. In the example shown, the difference between the outlet temperature and the ambient temperature is $T_{\mathrm{m}} - T_0 = 2\ \mathrm{K}$ for full-load operation and $25\ \mathrm{K}$ for part-load operation. The usable thermal energy in relation to the environment is therefore low for full-load operation, while it makes the difference in efficiency for part-load operation. This thermal energy is caused by dissipation and is only usable by an exergetic point of view but not from an energetic point of view.

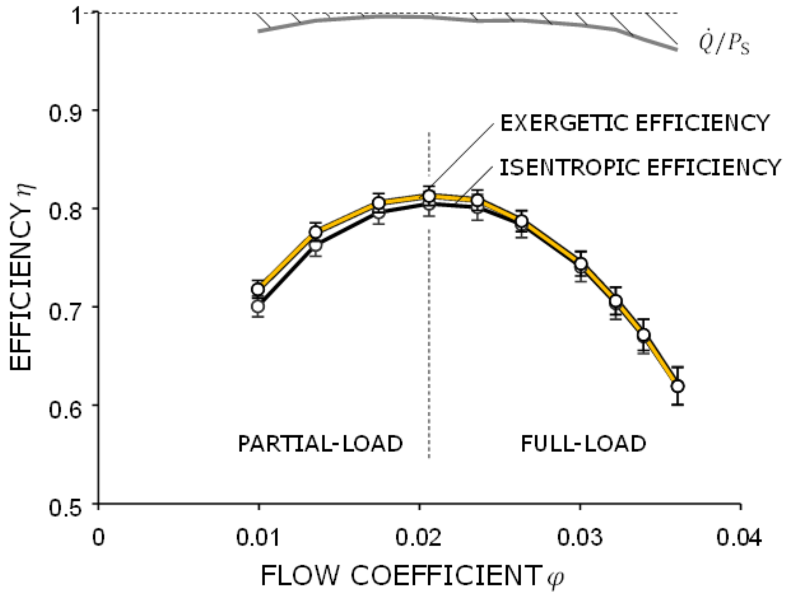

Figure 4 Exergetic efficiency compared to isentropic efficiency.
Hatched is the calculated power loss due to heat dissipation to the environment.

In addition, Figure 4 shows the proportion of heat dissipation to the environment calculated with equation (24) and verified with the thermal images. The dissipation has a minimum near the best efficiency point, and a maximum at full-load operation with a share of about 4 %. The heat transfer does not increase in full-load operation as a result of a higher temperature difference between the fan housing and the environment; this is just half as large as in partial-load operation. The reason for the higher heat transfer can be ascribed to the higher convection due to the air circulation in the area of the machine.

CONCLUSION

The first and second laws of thermodynamics are combined in the exergetic efficiency. As a result, its biggest benefit is thus its universal applicability. The exergetic efficiency does not only consider the fluid power from pressure build-up and delivery of a volume flow, but also the contained usable thermal energy to environment. The application of the exergetic efficiency to a high-pressure centrifugal fan has shown that only minor differences between exergetic and isentropic efficiency could be determined. Larger differences are to be expected with larger differences between outlet and ambient temperature. It has been shown that the exergetic efficiency can be determined with the test stands described in ISO 5801:2018. The measurement effort increases especially when measuring the outlet temperature. Here, sensors with low systematic uncertainty are required in order not to worsen the overall uncertainty. The determination of the temperature of the flow also plays a relevant role for the exergetic efficiency, which ought to be investigated more closely.

The thermal energy contained in the flow can be used to a certain extent. If the fan is used to ventilate buildings or tunnels, the surroundings are heated with the thermal energy. Since fans are also used for cooling processes, such as in air conditioning systems, the exergy contained in the exhaust air flow cannot always be used and can even be counterproductive. The exergetic efficiency applied to fans must therefore always be considered in the system. Such a system efficiency is becoming increasingly relevant today, as motivated in the introduction. For the evaluation of the component, on the other hand, the isentropic efficiency is recommended, as it has a high acceptability among users due to its simple applicability.

The next phase in the investigation will be to compare the diabatic and adiabatic machines.

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