EXTENDED RANKINE-BETZ THEORY FOR DESIGN OF TUNNEL VENTILATION SYSTEMS

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SUMMARY

Today, the system design of tunnel ventilation systems including jet fans neglects the stream tube contraction being covered by the Rankine-Betz theory. The presented system design tool extends in a truly physical manner the current method. The influence of the stream tube contraction is discussed and compared to the common but simplified design method by Meidinger dated back to 1964. In addition, the traffic is treated as a peristaltic flow at high Reynolds number and friction losses are modeled with common approaches. The new model allows analysis of several extreme situations, like normal traffic flow, traffic jam and fire in a tunnel. With the help of the new tool, tunnel design parameters (the number of jet fan units and the cross section ratio of jet fans and tunnel) and the operating conditions (velocity ratio of jet fan velocity and tunnel velocity) are predictable.

INTRODUCTION

Tunnel ventilations systems provide fresh air in a tunnel with different volume flow requirements. These requirements depend on the traffic and the situation, like normal traffic, traffic jams or an accident with fire. Estimations for number and size of jet fans in a tunnel are necessary during planning and design phase to meet all requirements. Computational fluid dynamic (CFD) is inefficient as a design tool due to long preparation and simulation times. Experimental investigations are expensive and effortful as well. Therefore, a fast and reliable prediction tool basing on analytic and generic models is preferable.

Today, the system design of tunnel ventilation systems including jet fans neglects the stream tube contraction being covered by the Rankine-Betz theory [1]. Neglecting the jet dynamics yields an analytic solvable method [2]. This simplified approach by Meidinger [2] is common and is used e.g. in the Swiss tunnel design guideline, ASTRA 13001 [3] or in Beyer's publication [4]. The motivation for our presented research emerges the remaining uncertainty due to model simplification.

The paper is organized as follows. The following section describes a tunnel ventilation system with a jet fans, followed by the modelling of a tunnel ventilation system and the losses. The results are presented and discussed. The paper closes with a summary and a conclusion.

TUNNEL VENTILATION SYSTEM

Figure 1 shows a tunnel ventilation system consisting of n similar jet fan units. The tunnel cross section A is constant. The actuator of one unit may be a single jet fan or several jet fans in parallel. The total cross section of the fans in one unit is denoted by a. The model captures air and traffic flow direction by the sign. Positive signs indicate the flow from left to right. Negative signs intends vice versa directions. The losses are inlet and outlet losses, friction losses, as well as mixing losses. The modelling of these losses is explained in detail in section *loss and traffic modelling*.

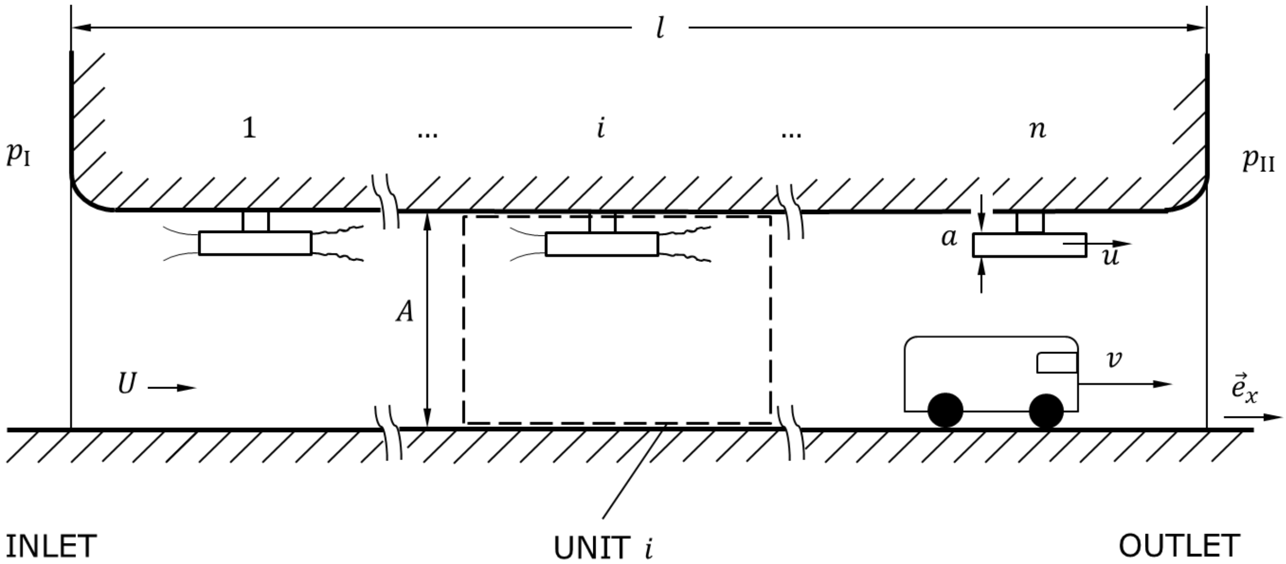

Figure 1: Tunnel ventilation system with jet fans and traffic.

On the one hand, the ventilation system has to provide a volume flow rate $\dot{V}$ depending on the traffic or an emergency. One the other hand the traffic causes a pressure rise varying in magnitudes depending on the flow velocity $U = \dot{V}/A$ and velocity v and density N/l of the traffic. N is the number of vehicles in the tunnel of length l.

Three different flow situations are important: normal traffic, traffic jam and the event of a fire. The event of a fire requires the highest amount of volume flow rate due to the remove of smoke and poisonous gases out of the tunnel.

The unknown and wanted design parameters are:

(i) the number of jet fan units n,
(ii) dimensionless axial velocity of the fan $\sigma := a/A$, and
(iii) dimensionless cross section of the jet fans $\mu := u/U$.

MODEL OF TUNNEL VENTILATION SYSTEM

Figure 2 a) shows one generic tunnel ventilation unit i (cf. Figure 1). The flow is incompressible, i.e. the density is constant $\varrho = const.$. In flow direction the pressure and velocity is to be calculated at five cross sections: at sections [0], [3] and [4] the pressure p_0, p_3, p_4 is constant across the tunnel cross section, since the curvature of the stream lines are small. This is not the case at sections [1] and [2]. Here, the pressure just at the inlet of the fan is denoted by p_1 and the pressure at the outlet of the fan is denoted by p_2.

Figure 2 b) shows the five control volumes (CV) necessary describing the flow. The inlet pressure can be set to zero $p_0 = 0$, since an incompressible flow is assumed and we count only the static pressure rise Δp. CV1 is for the bypass flow around the actuator. The stream tube up-stream the actuator is captured by CV2 with inlet cross section $\beta_1 A$. The outlet cross section of CV2 corresponds with the actuator cross section $a = \sigma A$. CV3 encloses the jet fan, i.e. the Rankine disk actuator. The pressure rise over the actuator is

$$\Delta p_\mathrm{A} = p_2 - p_1. \tag{1}$$

Hence, the fan shaft power for all fans of one unit is

$$P_\mathrm{S} = \frac{1}{\eta} \Delta p_\mathrm{A}\, u\, a = \frac{1}{\eta} \Delta p_\mathrm{A}\, U\, A\, \mu\, \sigma. \tag{2}$$

The fan efficiency is defined as usual $\eta \coloneqq \Delta p_\mathrm{A}\, u\, a / P_\mathrm{S} = 1 - P_\mathrm{l}/P_\mathrm{S}$ (dissipative power loss P_l and shaft power P_S).

u_S denotes the jet velocity at axial position [3]. In between sections [3] and [4] (CV5), the mixing of the jet and bypass flow takes place. Even though there is a pressure rise, i.e. $p_4 > p_3$, there are mixing losses being Carnot-type losses [5]. At the outlet of CV5 the flow is uniform with the velocity U.

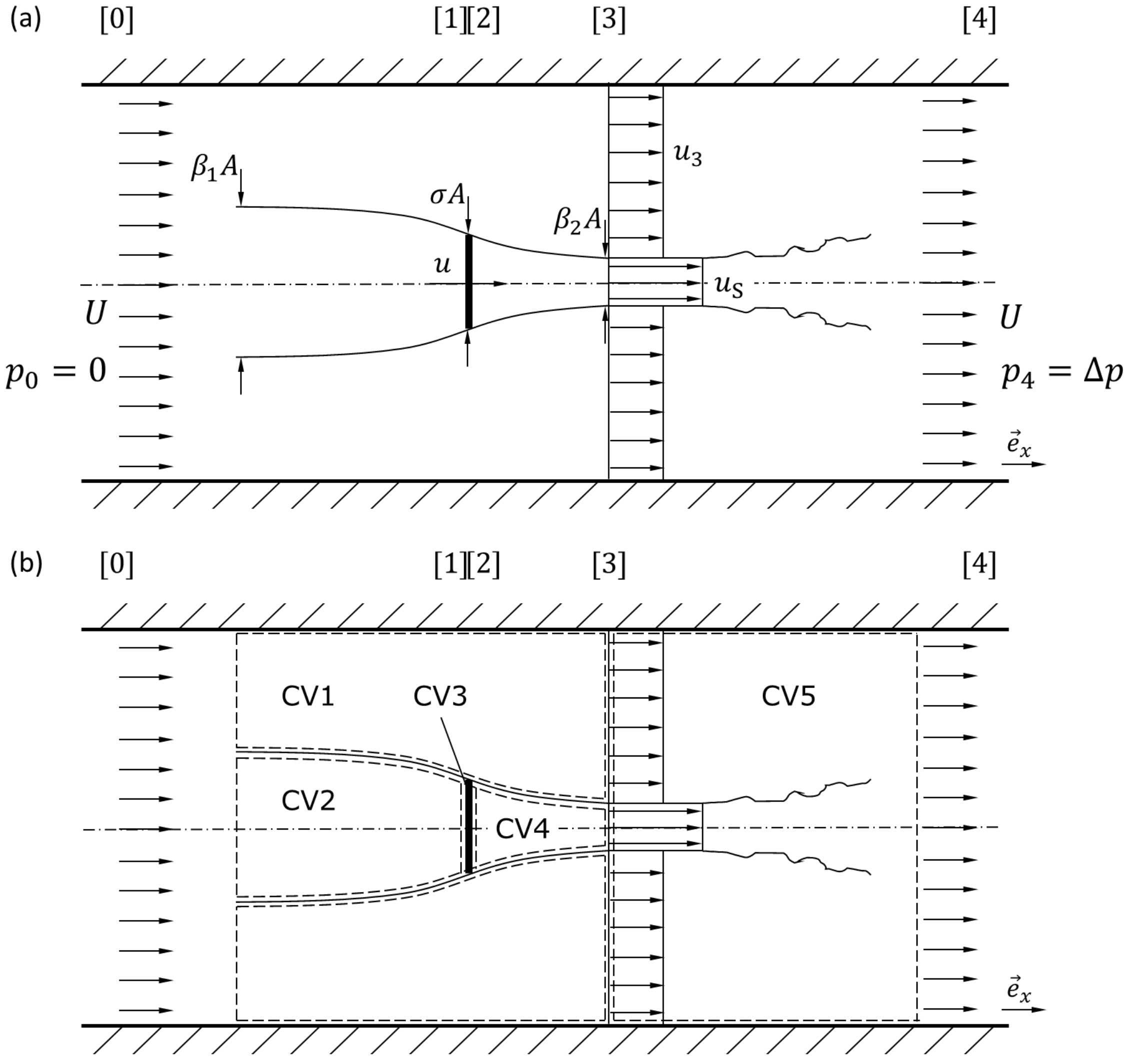

Figure 2: Schematic sketch of one ventilation unit: (a) velocities and cross sections. (b) Control volumes (CV).

The known parameters are the cross section of the tunnel A, the density ϱ as well as the required volume flow rate $\dot{V} = UA$. Therefore, the unknown parameters are the cross section ratios β_1, β_2 the thrust force T of the fan, the pressures p_1, p_2, p_3 and $p_4 = \Delta p$ and the velocities u_S and u_3. For those nine unknown parameters, we have nine independent equations listed in Table 1.

Table 1: System of equations describing one generic tunnel ventilation unit.

CV	domain	equation	
1	Bernoulli's equation 0 → 3	$p_3 = \frac{\varrho}{2}(U^2 - {u_3}^2)$	(3)
2	Bernoulli's equation 0 → 1	$p_1 = \frac{\varrho}{2}(U^2 - u^2)$	(4)
2	conservation of mass 0 → 1	$u = U\frac{\beta_1}{\sigma}$	(5)
3	momentum equation 1 → 2	$(p_2 - p_1)a = T$	(6)
4	Bernoulli's equation 2 → 3	$p_2 = p_3 + \frac{\varrho}{2}({u_\mathrm{S}}^2 - u^2)$	(7)
4	conservation of mass 2 → 3	$u_\mathrm{S} = u\frac{\sigma}{\beta_2}$	(8)
5	conservation of mass 3 → 4	$u_3(1 - \beta_2) + u_\mathrm{S}\beta_2 = U$	(9)
5	momentum equation 3 → 4	$\varrho {u_3}^2(1 - \beta_2) + \varrho {u_\mathrm{S}}^2\beta_2 - \varrho U^2 = \Delta p - p_3$	(10)
unit	momentum equation 0 → 4	$\Delta p A = T$	(11)

The system of equations (Table 1) is implemented in MATLAB and solved numerically using the MATLAB solver *fmincon*. The outcome of the model is conveniently given in a dimensionless form, i.e. pressure build up and efficiency. The pressure build up is only a function of σ and μ:

$$\psi \coloneqq \frac{2\Delta p}{\varrho U^2} = \psi(\sigma, \mu). \tag{12}$$

The efficiency of the tunnel ventilation unit is defined as usual as a dimensionless measure of the dissipation

$$\eta_\mathrm{u} \coloneqq \frac{\Delta p U A}{P_\mathrm{S}} = \frac{P_\mathrm{S} - P_\mathrm{l} - P_\mathrm{m}}{P_\mathrm{S}} = \eta - \frac{P_\mathrm{m}}{P_\mathrm{S}}, \tag{13}$$

with the power loss P_m due to mixing in the unit. Hence, the efficiency of the unit is always lower than the efficiency of the fan as expected. The shaft power $P_\mathrm{S} = \Delta p_\mathrm{A}\, u\, a/\eta = \Delta p_\mathrm{A}\, U\, A\, \mu\, \sigma/\eta$ and the efficiency of the unit yields

$$\eta_+ \coloneqq \frac{\eta_\mathrm{u}}{\eta} = \frac{\Delta p_\mathrm{A}\, u\, a}{\Delta p\, U\, A} = \frac{\psi_\mathrm{A}}{\psi}\frac{1}{\mu\sigma} = \eta_+(\mu) \tag{14}$$

with $\psi_\mathrm{A} \coloneqq 2\Delta p_\mathrm{A}/(\varrho U^2)$. As $\psi = \psi(\sigma, \mu)$ the efficiency η_+ is at a first glance a function of σ and μ. Later we will see, that η_+ is in fact only a function of μ.

The same is true for the limiting operating curve (c stands for critical)

$$\sigma_\mathrm{c} = \sigma_\mathrm{c}(\mu) \tag{15}$$

plotted in Figure 3. For $\sigma > \sigma_\mathrm{c}$ the bypass flow becomes unphysical, since the stream tube at the inlet expands until the cross section of the tunnel A is reached ($\beta_1 = 1$). This limit only exists, if the jet contraction is considered. The simplified model, e.g. Meidinger's model [2], does not show this limit due to neglecting the jet contraction. In typical tunnels the range of cross section ratios is limited to $\sigma = 0.005 \dots .0.02$, due to the presence of traffic.

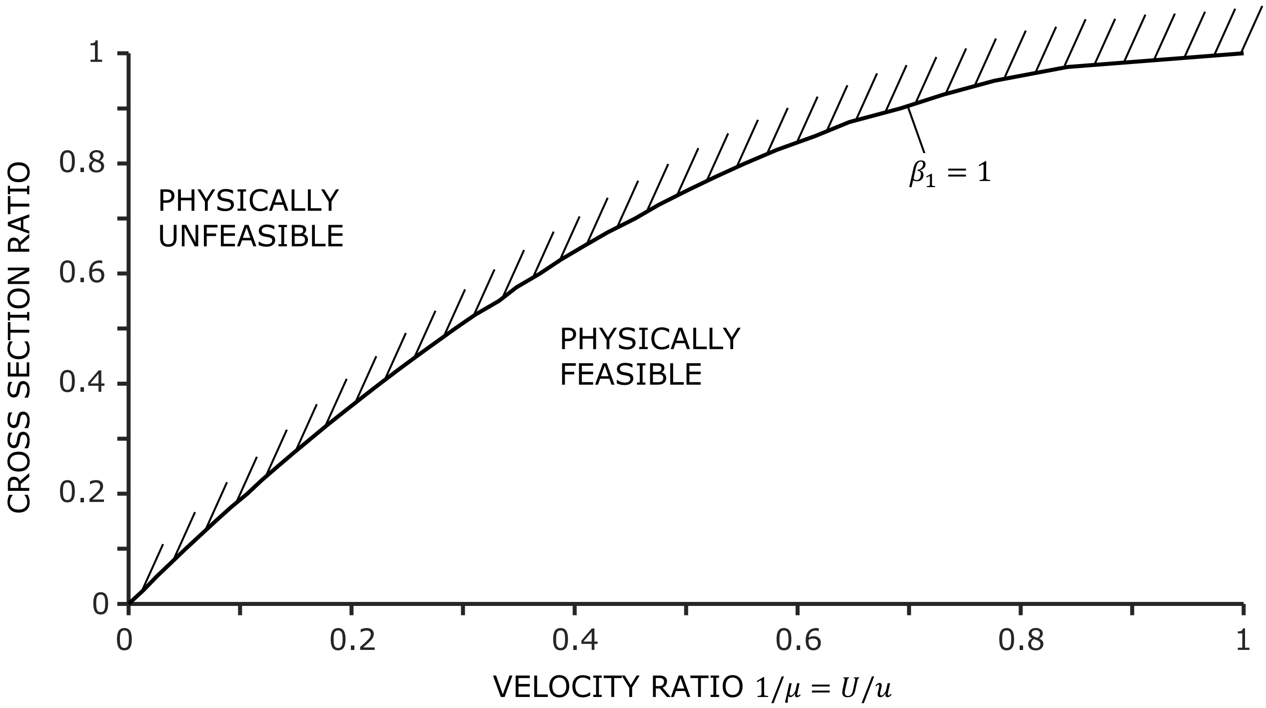

Figure 3: Physically feasible und unfeasible combinations of cross section ratio σ and velocity ratio $1/\mu$. Please note, typical cross section ratios for tunnel ventilation are $\sigma = 0.005 \dots 0.02$ and the velocity ratio is $1/\mu \approx 0.17$.

Figure 4 shows the pressure coefficient ψ versus the velocity ratio $1/\mu$ for four different cross section ratios σ. For increasing σ or increasing velocity ratio μ, the pressure coefficient ψ increases.

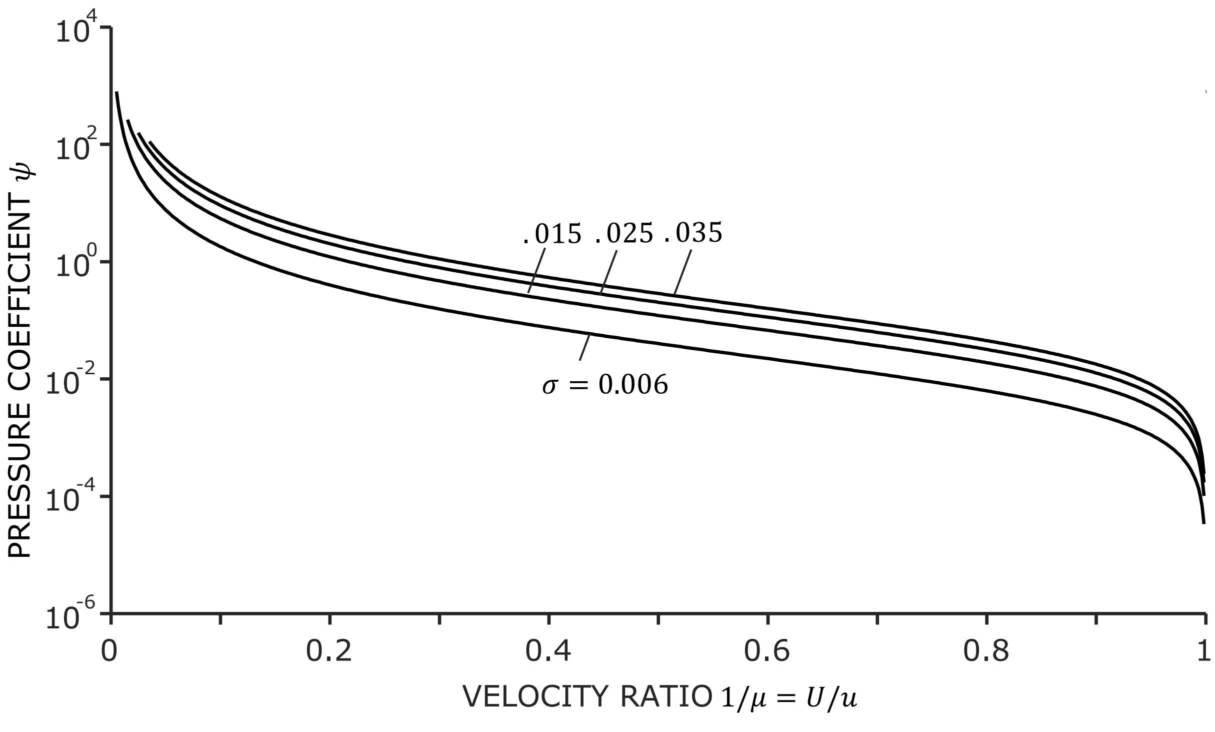

Figure 4: Pressure rise ψ versus velocity ratio μ for changing area ratios σ.

Figure 5 shows the pressure rise ψ of the model presented here and compared to Meidinger's model [2] for a constant cross section $\sigma = 0.006$. Even though the characteristics are similar, the presented and more physical model predicts about two times the pressure rise for the same operating conditions compared with Meidinger's prediction! It is important to emphasize: relying on Meidinger's model will result in oversized fans and hence unnecessary energy consumption and investment costs.

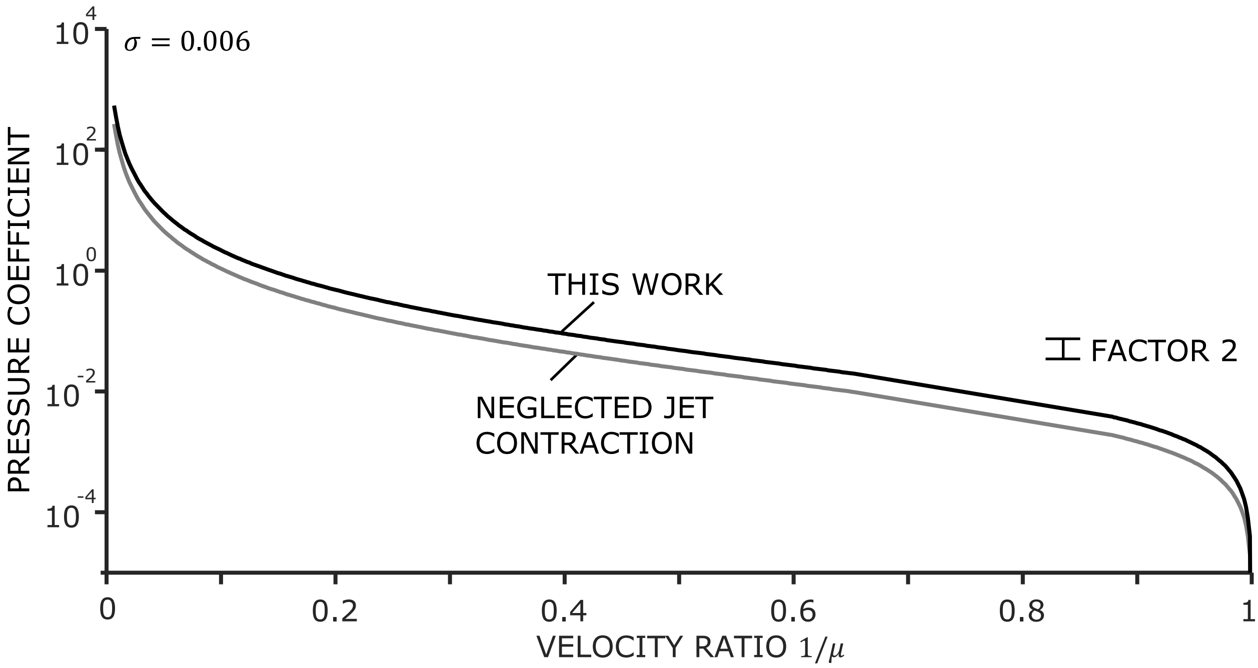

Figure 5: Comparison of total pressure rise ψ versus velocity ratio μ between the presented model and Meidinger's model. This model takes the contraction of the stream tube into account.

Figure 6 shows the efficiency of one unit η_+ versus the velocity ratio $1/\mu$. Meidinger's model predicts the efficiency as $\eta_+ = 2/(\mu + 1)$ and Darmstadt's model as $\eta_+ = 1/\mu$ for all cross sections ratios σ. Hence, neglecting jet contraction, as it is done currently, overpredicts the efficiency by up to 0.2 being not in line with the society need for energy efficiency of ventilation systems.

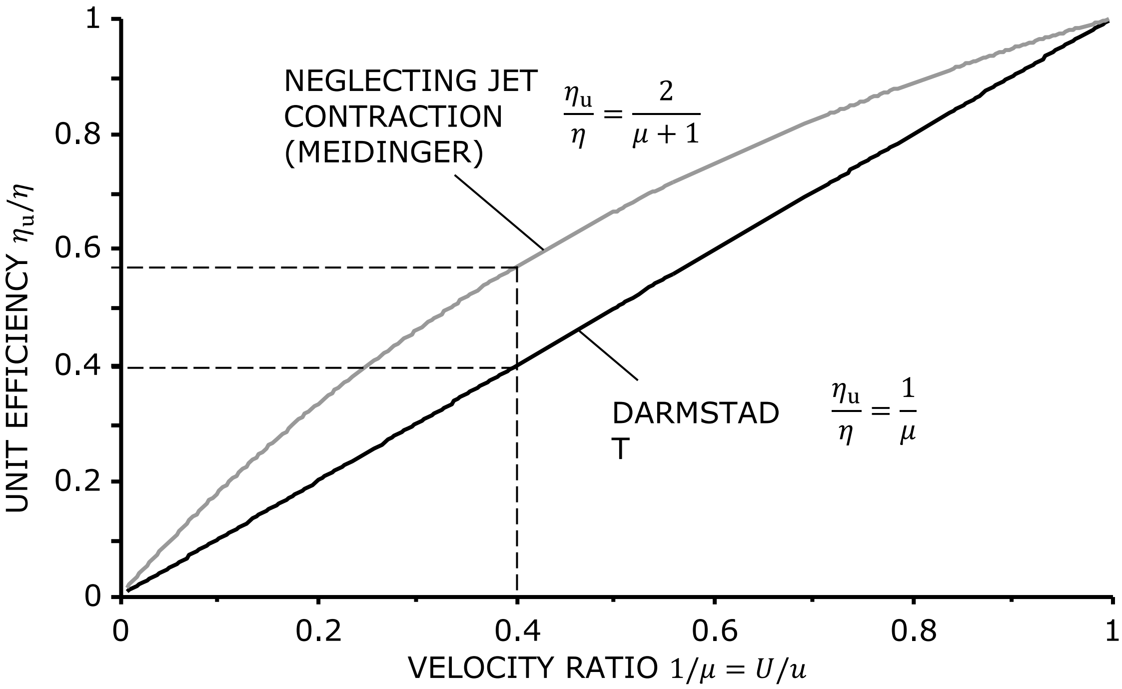

Figure 6: Unit efficiency η_u versus the velocity ratio μ for Meidinger's and Darmstadt's method for all cross section ratios σ.

LOSS AND TRAFFIC MODELLING

For quasi stationary flow, the pressure rise of the tunnel ventilation system balances the losses in the tunnel:

$$n\Delta p + \Delta p_{\mathrm{T}} = \Delta p_{\mathrm{L}} + p_{\mathrm{II}} - p_{\mathrm{I}}. \tag{16}$$

The traffic acts like a fan, causing a pressure rise Δp_{T}. Hence, Δp_{T} is on the left side of the equation. The difference of ambient pressure at tunnel inlet p_{I} and at tunnel outlet p_{II} (see Figure 1) is on the right hand side.

In turn, the pressure loss

$$\Delta p_{\mathrm{L}} = \Delta p_{\mathrm{f}} + \Delta p_{\mathrm{i}} + \Delta p_{\mathrm{o}} \tag{17}$$

is the sum of friction loss Δp_{f}, inlet loss Δp_{i} and outlet loss Δp_{o}. The losses are expressed as usual with dimensionless coefficients defining as

$$\zeta := \frac{\Delta p_{\mathrm{L}}}{\frac{\varrho}{2} U^2}, \tag{18}$$

with the pressure loss Δp_{L}. Eq. (18) can be rewritten with eq. (19) and yielding

$$\Delta p_{\mathrm{L}} = \frac{\varrho}{2} U^2 \left(\zeta_{\mathrm{f}} + \zeta_{\mathrm{i}} + \zeta_{\mathrm{o}}\right). \tag{19}$$

Friction losses are given by

$$\zeta_{\mathrm{f}} = \frac{\lambda\, l}{d}, \tag{20}$$

with the friction factor $\lambda = 0.015$ [3] and the diameter $d \propto \sqrt{A}$. Inlet and outlet losses are based on the Borda-Carnot loss. According to the Swiss tunnel guideline ASTRA 13001 [3] and the Austrian tunnel guideline ASFINAG [6], the loss coefficients are $\zeta_{\mathrm{i}} = 0.6$ for the inlet and $\zeta_{\mathrm{o}} = 1$ for the outlet.

The fan-like effect of the traffic is modelled by a peristaltic ansatz [7]. Figure 7 shows the traffic passing the tunnel und the detail shows the traffic simplified and transformed in the relative frame of reference.

The control volume moves with the velocity v and hence, the relative flow velocity is $w\vec{e}_x = U\vec{e}_x - v\vec{e}_x$ (unit vector $\vec{e}_x$ points from left to right). Depending on the sign and magnitude of U and v, the relative velocity w can be either positive or negative. For $w < 0$, the traffic supports the jet fans. For $w > 0$, the traffic works against the flow direction meaning the jet fans have to equal the losses and the pumping effect of the traffic.

The peristaltic ansatz for high Reynolds numbers uses Borda-Carnot shock losses: the sudden decrease of cross section ($A \to A'$), cf. Figure 7, causes a contraction in the restriction ($A' \to A''$) resulting in a Borda-Carnot shock losses. A second Borda-Carnot shock loss occurs at the sudden cross flow expansion ($A' \to A$). The pressure increase for traffic with N vehicles and high Reynolds numbers yields

$$\Delta p_{\mathrm{T}} = N(p_+ - p_-) = N\frac{\varrho}{2}|U - v|(U - v)\left(\frac{A - A''}{A}\right)^2. \tag{21}$$

According to the Swiss tunnel design guideline, ASTRA 13001 [3], the change in cross section can be written as

$$A - A'' = c_{\mathrm{d}} A_{\mathrm{v}}, \tag{22}$$

with the projected cross section of the vehicle A_v and the drag coefficient c_d considering the vehicle aerodynamics. The product of vehicle cross section and drag coefficient is $c_\mathrm{d}A_\mathrm{v} = 5.2\ \mathrm{m}^2$ for trucks and $c_\mathrm{d}A_\mathrm{v} = 0.9\ \mathrm{m}^2$ for cars [3]. Hence, we have

$$\psi_\mathrm{T} \coloneqq \frac{2\Delta p_\mathrm{T}}{\varrho U^2} = N\left|1-\frac{v}{U}\right|\left(1-\frac{v}{U}\right)\left(c_\mathrm{d}\frac{A_\mathrm{v}}{A}\right)^2. \tag{23}$$

$N, v/U, c_\mathrm{d}A_\mathrm{v}/A$ are the independent dimensionless products describing the traffic flow.

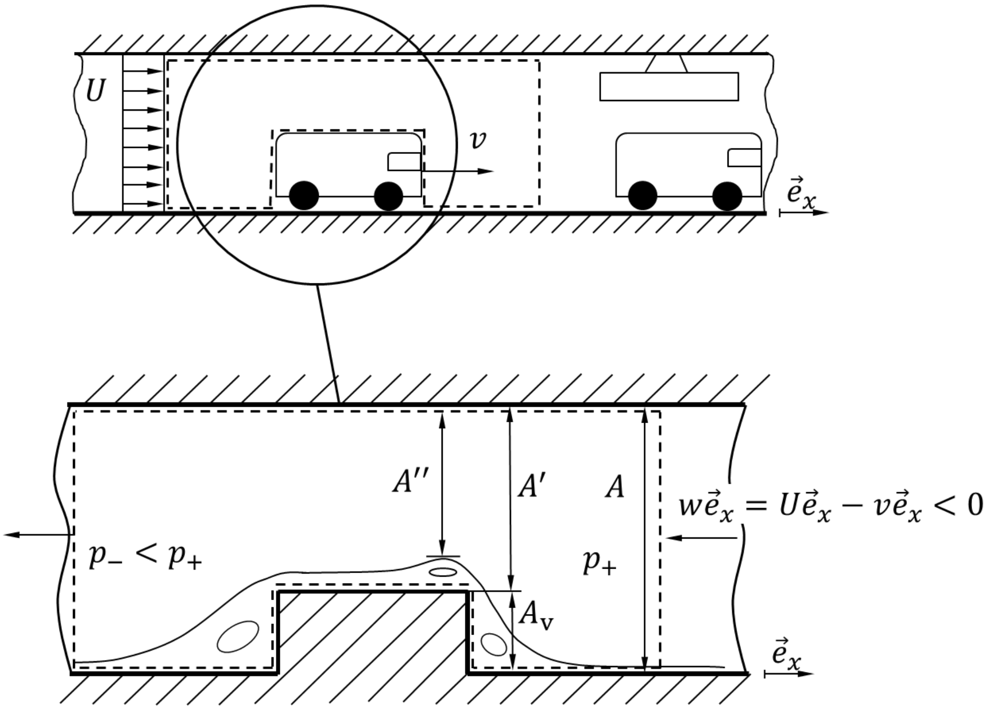

Figure 7: Traffic modelling with a peristaltic ansatz [7].

RESULTS

For the presentation and discussion of the design tool, an alternative form is useful. For clarification, an effective loss coefficient is defined as

$$\zeta_\mathrm{E} \coloneqq \zeta_\mathrm{f} + \zeta_\mathrm{i} + \zeta_\mathrm{o} - \psi_\mathrm{T} + \psi_\mathrm{I,II}, \tag{24}$$

with $\psi_\mathrm{I,II} \coloneqq 2(p_\mathrm{II} - p_\mathrm{I})/(\varrho U^2)$. ψ_T and $\psi_\mathrm{I,II}$ can be positive or negative. In constrast $\zeta_\mathrm{f}, \zeta_\mathrm{i}, \zeta_\mathrm{o}$ are in any case positive. With the abbreviation (24), the pressure balance, eq. (17), is rewritten in the equivalent, short and dimensionless form

$$\frac{n}{\zeta_\mathrm{E}} = \frac{1}{\psi_\mathrm{A}}. \tag{25}$$

The required pressure rise of one tunnel ventilation unit Δp_A does not change the solution of the model of one tunnel ventilation unit but it influences the number n of them.

Figure 8 shows the number of tunnel ventilation units n/ζ_E versus the ratio of volume flow rates $\dot{V}_\mathrm{A}/\dot{V} = \sigma\mu$ for different cross section ratios σ. The design tool gives results for all cross flow ratios for a given operating point.

How to use Figure 8 is explained by an example with a cross section ratio $\sigma = 0.015$: a given operating point $\dot{V}_\mathrm{A}$ of the jet fans and the required volume flow rate through the tunnel $\dot{V}$ determine the volume flow ratio $\dot{V}_\mathrm{A}/\dot{V} = \sigma\mu$. On the one hand, the aim is an efficient system meaning the cross section ratio σ has to be as large as possible but on the other hand, this leads to an increase in required units n. The number of tunnel ventilation units n/ζ_E are determinable on the ordinate depending on the effective losses of the tunnel ζ_E. Additionally, an optimized flow channel with respect to the effective losses ζ_E shall be aspired to reduce the number of jet fan units n.

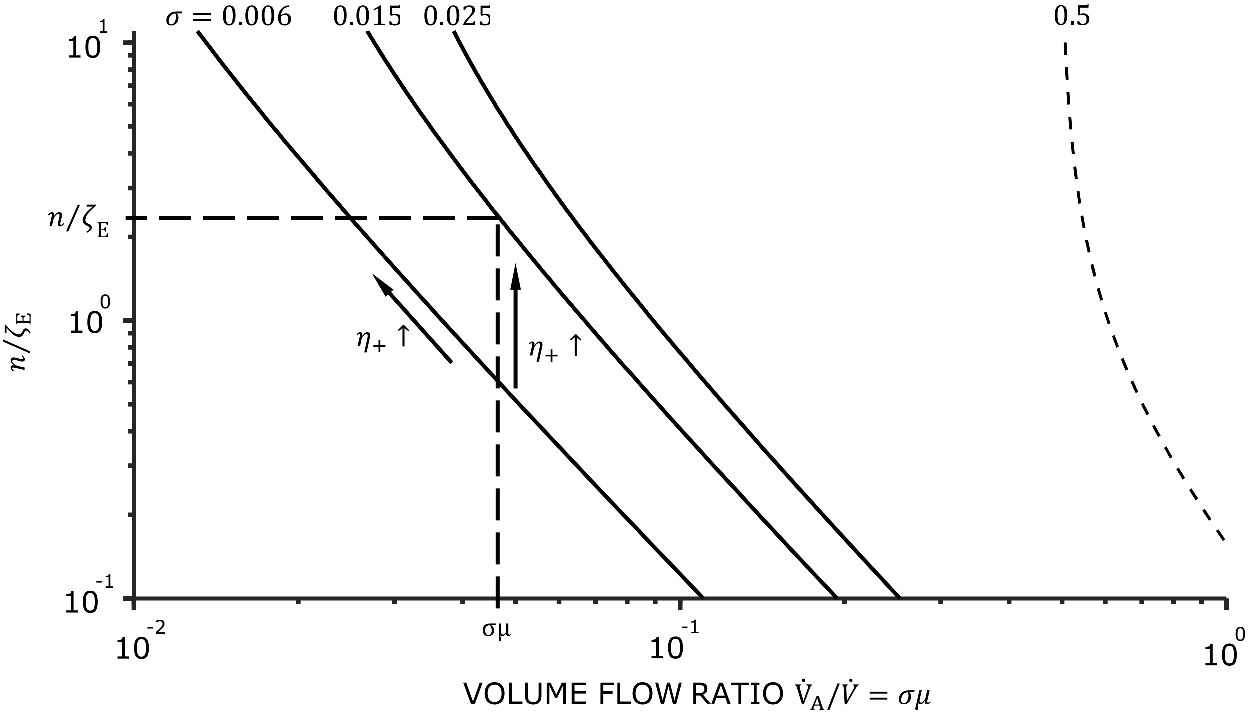

Figure 8: Number of jet fan units per loss coefficient n/ζ_E versus volume flow ratio $\dot{V}_A/\dot{V} = \sigma\mu$ with the cross section ratio σ as independent variables.

DISCUSSION

The model presented here shows good results in the physical plausible range of velocity ratios μ. The characteristic curves (Figure 4, Figure 5 and Figure 6) show large deviations between Darmstadt's and Meidinger's simplified method.

The axial positions (0, 1, 2, 3 and 4) marked in Figure 9 are in agreement with Figure 2. The black lines indicates the pressure curves of Darmstadt's model and the grey lines show the pressure curves of Meidinger's model. In addition, all pressures with a prime (') indicate a pressure referring to Meidinger's model. The acceleration of the flow up-stream the actuator ($0 \rightarrow 1$) is identical for both models. The pressure rise over the actuator Δp_A ($1 \rightarrow 2$) has to be different due to the stream tube expansion of the bypass flow ($0 \rightarrow 3$). Therefore, the bypass flow is decelerated and the pressure increases. Meidinger's model does not consider the stream tube and hence, the pressure at actuator outlet is the pressure at mixing zone inlet $p'_2 = p'_3$. The presented model takes the contraction of the stream tube into account and thus the flow is accelerated ($2 \rightarrow 3$). I.e. Darmstadt's model predicts a higher pressure increase over the actuator Δp_A and results in a higher pressure rise of the tunnel ventilation unit with a lower efficiency of the unit.

Summarized, three reasons explain the before mentioned offset in pressure coefficient ψ of both models (Figure 5). First, the bypass flow expansion generates a pressure increase $p_3 > p'_3$. Secondly, the jet contraction down-stream the actuator increases the pressure rise of the actuator $\Delta p_\mathrm{A} > \Delta p'_\mathrm{A}$, hence the model presented here predicts a higher power input[1]. Thirdly, the increased

[1] The pressure rise of the actuator is $\Delta p_\mathrm{A} = p_2 - p_1$ for Darmstadt's model and $\Delta p'_\mathrm{A} = p'_2 - p'_1$ for Meidinger's model.

stream velocity causes a higher pressure rise in the mixing zone (3 → 4) [5] which is coupled with a lower efficiency due to higher mixing losses.

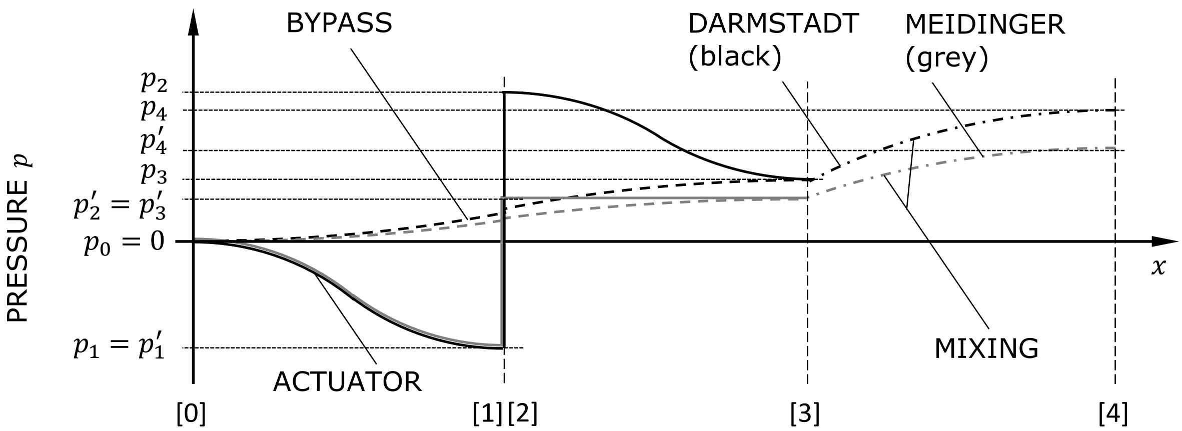

Figure 9: Qualitative characteristic of static pressure through the tunnel ventilation unit.

CONCLUSION

This paper presents an extended Rankine-Betz model for the design of tunnel ventilation systems. For the first time, the stream tube contraction up- and down-stream the jet fan is considered. The non-linear system of equations is solved numerically. The number of necessary jet fan units depends on the effective tunnel losses ζ_{E}, cross section ratio σ and velocity ratio μ. Furthermore, Darmstadt's model is compared to a simplified model by Meidinger [2], which neglects the stream tube contraction. The model presented here in general leads to higher pressure coefficients ψ and lower efficiencies η of the ventilation unit.

A jet boundary layer along a wall may easily be integrated into the model taking dissipations due to jet streams near the wall into account [8, 9]. In the future, the presented model shall be validated with experimental data regarding the number of tunnel ventilation units and the operating point of the installed jet fans.

The presented model is not limited to tunnel applications for trucks, cars and railways. It can be used for more complex systems like ventilation systems of mines or subways. Furthermore, arrays of jet fans are imaginable as well.

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