TECHNICAL OPERATIONS RESEARCH (TOR) - ALGORITHMS, NOT ENGINEERS, DESIGN OPTIMAL ENERGY EFFICIENT AND RESILIENT COOLING SYSTEMS

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SUMMARY

By considering not only single components, but also their interplay, the overall energy efficiency of a ventilation system can be considerably improved. A design method from this system point of view, the method TOR ("Technical Operations Research"), is presented in this work. With TOR, we can algorithmically search and evaluate the energy efficiency of all possible system layouts of different fans, and find the global optimal system design. Therefore, we employ mathematical methods from the field of discrete optimization. We show the possibilities of this systematic design approach, and design a ventilation system for buildings that is energy efficient and resilient.

INTRODUCTION

Currently, ventilation systems in buildings are often designed manually and the selection of components and the system layout depend on the designer's experience. If three different system designers get the same design task, the resulting system configurations will most likely vary, meaning that at least one of them cannot be optimal. To find the provably global optimal system topology, a systematic approach that allows to assess all possible system layouts is needed. Moreover, designers of ventilation systems often only consider the maximum 'worst-case' loading condition. However, this maximum load case has only to be fulfilled in a very short period of the actual usage time.

It was shown in [1] that it is often possible to improve the overall energy efficiency of a system by taking more possible design choices into account, especially if the system also runs in partial load. To systematically model and optimize technical systems, the method TOR ("Technical Operations Research") [2] was developed at the chair of fluid systems at TU Darmstadt.

This method uses mathematical optimization techniques to build energy efficient technical systems with the help of algorithms. The method TOR bases on seven steps which are shown in Figure 1:

Figure 1: The TOR pyramid [2].

In the first step, the function of the system to be planned has to be determined. This function is given by the load requirements the system will have to fulfill. In our approach, we not only consider the 'worst-case' load condition, but a load profile of different load scenarios and their respective frequency.

In the second step of the TOR pyramid, the goal of the optimization is specified. This goal is subjective and can even be multi-criterial. In our case, we want to find the ventilation system with the lowest power consumption and still guarantee system resilience. In the underlying mathematical optimization program, this goal corresponds to the objective function that is either maximized or minimized.

In the third step, the playing field of the optimization has to be specified, i.e., the degrees of freedom of the optimization. In this step, a construction kit of fans is specified, out of which the optimization algorithm can choose and build an optimal system topology. If we want to find the global optimal solution given a specific construction kit, it is mandatory to specify the entirety of all possible choices.

The computation of the most efficient system is done in step four. All potential layouts have already been integrated in the model and in this step a mathematical optimization algorithm is used to retrieve the most promising choice.

In step five this optimal solution is verified by using 0D-models and simulation environments like Modelica [3]. A key element of evolving new systems is the validation step. In this step, the optimal solution is validated with experimental data derived from test rigs. In the final step, the optimal layout is built, if the solution passes the validation.

FUNDAMENTALS OF RESILIENCE

Next to the energy efficiency, the overall system availability is often a key factor when assessing different layout choices. Therefore, we also consider the resilience of the derived system. In this section, we develop the fundamentals to understand resilience in technical systems.

The concept of resilience was first used in psychology [4]. Nevertheless, it is also very promising to transfer this concept to engineering. In general, resilience in the engineering domain describes the possibility of a technical system to withstand temporary severe failures and recover from theses failures [5]. Further, in comparison to robust systems, resilient systems allow a minimal function in the case of an occurring malfunction in one or more parts of the whole system. Robust systems on the other side withstand several predefined input variations, e.g. different loads on a mechanical structure [6]. In the case of a malfunction of one part, they usually cannot guarantee a minimal function.

Two questions are important for understanding the development of resilient systems. The first is which possibilities do exist to enhance the resilience. The second is how to measure the resilience of a technical system. In general, there exist different ways to improve a system's resilience. One possibility is the usage of redundant components. Additionally, a technical system which is able to detect failures and react has a higher possibility to withstand these failures. This ability of changing the internal state is another, more high level manifestation of a system's resilience. However, the considerations about enhancing a system's resilience are only valuable, if it is possible to measure resilience. One possibility for a resilience measure is the k-reliability of a technical system. If a system is k-reliable, it guarantees a predefined minimum function despite the failure of k arbitrary components.

VENTILATION SYSTEM

The ventilation system in this paper is based on the work of Schänzle et al. [2]. We consider a ventilation system for cooling in an office with two bureaus and one conference room. The calculus of the relevant pressure and volume flow is conducted with VDI 2078 [7]. Consistent to [2] we use three load profiles in this paper to fulfill the requirements of a cooling system for an office with bureaus and one conference room. They are shown in Table 1.

Table 1: Load profiles of two load scenarios.

Scenario Sc	**Time portion** τ_{sc}	**Pressure [Pa]** Δp_{sc}^{Sc}	**Volume flow [m³/h]** $\dot{V}_{sc}^{Sc}$
1	55 %	150	6200
2	30 %	175	9300
3	15 %	200	12400

While defining the function of the system by these load scenarios is the first step of the TOR method, the next step is to specify the playing field. In this application example, we want to make sure that the load cases can be fulfilled by using more than one fan, and consider all parallel connections that can be built by a given construction kit of fans.

For modelling a high variety of different fans within our construction kit, we use a dimensionless representation of a series of fans and derive the characteristic curves of the individual fans of this series by scaling laws. Therefore, we use the dimensionless coefficients for pressure (ψ), efficiency

(η) and power (λ). They are all highly non-linear functions of the dimensionless flow coefficient (φ). The definitions of Equation (1) to (4) are used in this paper (cf. [8]).

$$\Delta p = \frac{\pi^2}{2}\psi\rho n^2 d^2 \tag{1}$$

$$\dot{V} = \frac{\pi^2}{4}\varphi n d^3 \tag{2}$$

$$P = \frac{\pi^4}{8}\lambda\rho n^3 d^5 \tag{3}$$

$$\lambda = \frac{\psi\varphi}{\eta} \tag{4}$$

The best efficiency point is scaled up by formula (5) (cf. [2]) and used in the playing field.

$$\Delta\eta = \frac{1}{5}(1-\eta_{\mathrm{m}})\frac{\Delta Re}{Re_{\mathrm{m}}} \tag{5}$$

Here Re_m and η_m describe the Reynolds number and the efficiency of the model system. The model describes the fan, which was measured on a test rig and used as a basis for the considered series.

MATHEMATICAL OPTIMIZATION

In this work, we use mathematical optimization to derive the global optimal solution algorithmically. In this Section, we give the necessary mathematical basics.

Each optimization program consists of a set of variables, and an objective function. While the variables represent unknown quantities, the objective function defines the goal of the optimization. In constrained optimization programs, the variables of the optimization program are additionally restricted to a set of feasible values by constraints that have to be satisfied. In our case, these constraints are given by linear and non-linear functions and model the playing field of our optimization. The goal of the optimization is to find feasible values for the variables that i) satisfy all constraints and ii) minimize or maximize the objective function.

In this paper, we want to optimize the ventilation system regarding two goals. The first one is the energy efficiency of the system. The second one is to enhance the resilience of the whole system. We therefore have a multi-objective optimization task. Since Equations (1) – (5) are non-linear functions, we use a non-linear approach to model our system. Another approach would be to piecewise linearize the non-linear equations and use algorithms from the field of linear optimization. However, [9] shows that in the case of highly non-linear functions, non-linear modeling combined with non-linear optimization has benefits in comparison to piecewise linear models and linear optimization regarding the solving time.

The resulting optimization program is a Mixed-Integer Non-Linear Program (MINLP). An example for a MINLP is shown in Equation (6).

$$\begin{aligned} &\text{minimize}\, f(\vec{x}) \\ &\text{subject to}\;\; g_i(\vec{x}) = 0, i \in \mathbb{Z} \\ &\vec{x} \geq 0 \\ &\vec{x} \in \{\mathbb{R}, \mathbb{Z}\} \end{aligned} \tag{6}$$

In this case, the objective function $f(\vec{x})$ is minimized. Without loss of generality, the constraints can be described by $g_i(\vec{x}) = 0$, since inequality restrictions can be transformed into equality restrictions with the help of additional variables. The decision variables $\vec{x}$ consist of continuous and integer variables and are conventionally defined as positive values. The integer variables describe discrete decisions, like the possibility to buy a number of specific fans or not. The continuous variables are used to model the physical behavior of the system. Continuous variables are for example the pressure (Δp), the volume flow $(\dot{V})$, or the power consumption (P). In general, non-linear programs, like shown in Equation (6), are non-convex. This means, that they have multiple local minima and it is very challenging to find the global minimum. In this paper, we use SCIP [10] to solve the underlying non-linear model. SCIP is a solver designed especially to solve non-convex non-linear mixed-integer optimization programs. We model our optimization problem in Python and use PySCIPOpt [11] to call SCIP from Python. In the next section, we describe the underlying mathematical model to build an optimized fan system for cooling an office building

OPTIMIZATION MODEL

As mentioned earlier, the model consists of different continuous and integer variables. These variables define the basis of the following optimization. Therefore, all variables and their respective domain are explained in Table 2.

Table 2: Variables of the underlying MINLP.

Variable	Description	Domain	Unit
P	Power	$\mathbb{R}^+$	W
λ	Dimensionless power coefficient	[0.1,0.5]	−
φ	Dimensionless flow coefficient	[0.1,1]	−
η	Dimensionless efficiency coefficient	[0,1]	−
η^{ref}	Scaled efficiency	[0,1]	−
η^{norm}	Normalized efficiency of the model	[0,1]	−
n	Rotational speed	$[3, 35]$	1/s

The variables can assume different values in each load case and for each fan in the predefined construction kit. These values are set by the optimization algorithm. The best feasible value configuration represents the global optimal solution which corresponds to the system layout and control. Next to these variables, constant parameters are also defined in the model. Primarily the required volume flow and pressure in the fan system which are specified in the predefined load cases in Table 1. Next to these values, we use a model fan as the basis of the computation. Therefore, we use the model parameters η_m, n_m and d_m to describe the efficiency, rotational speed and diameter of the model respectively. For the computation we further need the air density ρ.

Table 3: Constants.

Constant	Description	Value	Unit
$\dot{V}$	Volume flow	cf. Table 1	m³/s
Δp	Pressure	cf. Table 1	Pa
η_{m}	Efficiency of the model fan	0.74	−
n_{m}	Rotational speed of the model fan	20.0	1/s
d_{m}	Diameter of the model fan	0.63	m
ρ	Air density	1.2041	kg/m³

Equations (1) – (4) are highly non-linear. For example, in Equation (3), the diameter of each fan has the power five. These highly non-linear equations can be transformed to enable a faster solving process: The choice of fan diameters is modeled as a set of N discrete values, and the different diameters are activated with binary variables. Like this, one fan with different diameters is represented by N equations (one for each possible diameter) in which the diameter is transformed into a constant. With this model improvement, it is possible to define and solve a Mixed-Integer Non-Linear Program, and to optimize the energy-efficiency and resilience of the whole system.

Figure 2 shows the construction kit of the optimization. Four different diameters are considered in the optimization model. The playing field consists of two fans of each size to enable a redundant usage of different fans. Overall, this results in a playing field of eight fans in total. With this playing field, it is e.g. possible to choose only one fan as well as up to eight parallel fans to fulfill the load cases of Table 1.

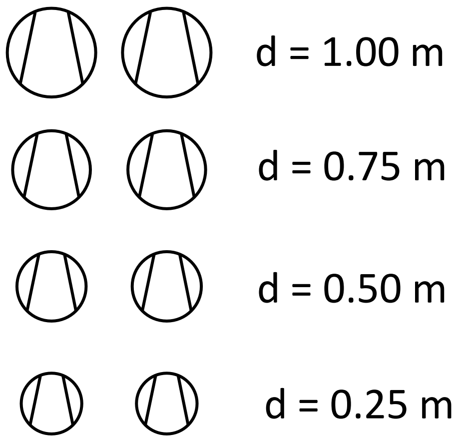

Figure 2: Used fan diameters.

As mentioned earlier, we use the characteristic curves of a model fan and use the relationship of Equation (1) - (4) to compute the dimensionless coefficients. We derive two curves to describe a product line of this fan. The first one is the normalized efficiency η^{norm} as a function of φ, the second one is the dimensionless power coefficient λ as a function of φ. We use a quadratic least-square fit to model η^{norm}as a function of φ.

$$\eta^{\mathrm{norm}} = \alpha_1(\varphi - \varphi^{\mathrm{max}})^2 + 1 \tag{7}$$

In Equation (7) the efficiency curve is equivalent to a parabola with a negative factor α_1. The maximum value $\eta^{\mathrm{norm}}(\varphi) = 1$ is reached at the value φ^{max}. Next to this, we use a cubic least-square fit for the power coefficient λ, cf. Equation (8).

$$\lambda = \alpha_2\varphi^3 + \alpha_3\varphi^2 + \alpha_4\varphi + \alpha_5 \tag{8}$$

In this equation, only the first coefficient α_2 is negative. All other coefficients are positive. The model approximates the values for λ in the range between 0.1 and 0.5.

In addition to these equations, binary variables are used to model the decisions to use a specific fan in a given load scenario. The whole optimization model, which is used to develop a system with multiple fans, is shown in Equation (9.1) to (9.14). The objective (cf. Equation (6)) has an important role in optimization programs, since it defines the goal of the designer. In our case, we want to minimize the sum of the power consumption of all fans in the system. Sc represents the set of all load scenarios. $\mathcal{V}$ is the set of all possible fans. It applies for all Equations $\forall v \in \mathcal{V}, \forall sc \in Sc$, if not stated otherwise. While Equations (9.1) – (9.14) represent a mathematical model which is used to find the most energy efficient system structure, we can also add additional constraints in order to aim for an energy efficient and resilient system. These additional constraints are shown in Equation (10.1) – (10.17). The modified objective is shown in Equation (11). It is used instead of the objective (9.1). To enable valid solutions, we use an additional continuous variable Z to allow a lower volume flow as desired. This difference between the total possible volume flow in case of a malfunction and the desired volume flow of Table 1 is minimized in the modified objective. The index R describes additional variables for the resilience computations.

$$\text{Minimize } \sum_{v\in\mathcal{V}} \sum_{sc\in Sc} \tau_{sc} P_{v,sc} \tag{9.1}$$

subject to

$$\Delta p_{v,sc} - \Delta p_{sc}^{\mathrm{Sc}} \leq \left(1 - k_{v,sc}\right) p^{\mathrm{max}} \tag{9.2}$$

$$\Delta p_{v,sc} - \Delta p_{sc}^{\mathrm{Sc}} \geq -\left(1 - k_{v,sc}\right) p^{\mathrm{max}} \tag{9.3}$$

$$\Delta p_{v,sc} \leq k_{v,sc} p^{\mathrm{max}} \tag{9.4}$$

$$\varphi_{v,sc} \geq 0.1 k_{v,sc} \tag{9.5}$$

$$\dot{V}_{v,sc} \leq \dot{V}^{\mathrm{max}} k_{v,sc} \tag{9.6}$$

$$\sum_{v\in\mathcal{V}} \dot{V}_{v,sc} = \dot{V}_{sc}^{\mathrm{Sc}} \ \forall\ sc \in Sc \tag{9.7}$$

$$P_{v,sc} = \frac{\pi^4}{8} \lambda_{v,sc} \rho n_{v,sc}^3 d_v^5 \tag{9.8}$$

$$\dot{V}_{v,sc} = \frac{\pi^2}{4} \varphi_{v,sc} n_{v,sc} d_v^3 \tag{9.9}$$

$$\Delta p_{v,sc} \varphi_{v,sc} = \frac{\pi^2}{2} \lambda_{v,sc} \eta_{v,sc} \rho n_{v,sc}^2 d_v^2 \tag{9.10}$$

$$\eta^{\mathrm{ref}}_{v,sc} = \eta_{\mathrm{m}} + \frac{1}{5}(1-\eta_{\mathrm{m}})\left(\frac{n_{v,sc}d_v^2}{n_{\mathrm{m}}d_{\mathrm{m}}^2} - 1\right) \tag{9.11}$$

$$\eta_{v,sc} = \eta^{\mathrm{norm}}_{v,sc}\eta^{\mathrm{ref}}_{v,sc} \tag{9.12}$$

$$\eta^{\mathrm{norm}}_{v,sc} = \alpha_1(\varphi_{v,sc} - \varphi^{\mathrm{max}})^2 + 1 \tag{9.13}$$

$$\lambda_{v,sc} = \alpha_2\varphi^3_{v,sc} + \alpha_3\varphi^2_{v,sc} + \alpha_3\varphi_{v,sc} + \alpha_4 \tag{9.14}$$

$$\Delta p^{\mathrm{R}}_{v,sc} - \Delta p^{\mathrm{Sc}}_{sc} \leq \left(1 - k^{\mathrm{R}}_{v,sc}\right)p^{\mathrm{max}} \tag{10.1}$$

$$\Delta p^{\mathrm{R}}_{v,sc} - \Delta p^{\mathrm{Sc}}_{sc} \geq -\left(1 - k^{\mathrm{R}}_{v,sc}\right)p^{\mathrm{max}} \tag{10.2}$$

$$\varphi^{\mathrm{R}}_{v,sc} \geq 0.1\, k^{\mathrm{R}}_{v,sc} \tag{10.3}$$

$$\dot{V}^{\mathrm{R}}_{v,sc} \leq \dot{V}^{\mathrm{max}}k^{\mathrm{R}}_{v,sc} \tag{10.4}$$

$$\sum_{v\in\mathcal{V}} \dot{V}^{\mathrm{R}}_{v,sc} + \epsilon^{\mathrm{R}}_{v,sc} = \dot{V}^{\mathrm{Sc}}_{sc}\ \forall\ sc \in Sc \tag{10.5}$$

$$P^{\mathrm{R}}_{v,sc} = \frac{\pi^4}{8}\lambda^{\mathrm{R}}_{v,sc}\rho {n^{\mathrm{R}}}^3_{v,sc}d_v^5 \tag{10.6}$$

$$\dot{V}^{\mathrm{R}}_{v,sc} = \frac{\pi^2}{4}\varphi^{\mathrm{R}}_{v,sc}{n^{\mathrm{R}}}_{v,sc}d_v^3\ \forall \tag{10.7}$$

$$\Delta p^{\mathrm{R}}_{v,sc}\varphi^{\mathrm{R}}_{v,sc} = \frac{\pi^2}{2}\lambda^{\mathrm{R}}_{v,sc}\eta_{v,sc}\rho {n^{\mathrm{R}}}^2_{v,sc}\, d_v^2 \tag{10.8}$$

$$\eta^{\mathrm{R,ref}}_{v,sc} = \eta_{\mathrm{m}} + \frac{1}{5}(1-\eta_{\mathrm{m}})\left(\frac{{n^{\mathrm{R}}}_{v,sc}d_v^2}{n_{\mathrm{m}}{d_{\mathrm{m}}}^2} - 1\right) \tag{10.9}$$

$$\eta^{\mathrm{R}}_{v,sc} = \eta^{\mathrm{R,norm}}_{v,sc}\eta^{\mathrm{R,ref}}_{\mathrm{v,sc}} \tag{10.10}$$

$$\eta^{\mathrm{R,norm}}_{v,sc} = \alpha_1(\varphi^{\mathrm{R}}_{v,sc} - \varphi^{\mathrm{max}})^2 + 1 \tag{10.11}$$

$$\lambda^{\mathrm{R}}_{v,sc} = \alpha_2{\varphi^{\mathrm{R}}_{v,sc}}^3 + \alpha_3{\varphi^{\mathrm{R}}_{v,sc}}^2 + \alpha_3\varphi^{\mathrm{R}}_{v,sc} + \alpha_4 \tag{10.12}$$

$$k^{\mathrm{R}}_{v,sc} \leq k^{\mathrm{buy}}_v \tag{10.13}$$

$$k^{\mathrm{buy}}_v \geq k_{v,sc} \tag{10.14}$$

$$\sum_{v\in\mathcal{V}} k^{\mathrm{R}}_{v,sc} \leq \sum_{v\in\mathcal{V}} k^{\mathrm{buy}}_v - 1 \tag{10.15}$$

$$\epsilon^{\mathrm{R}}_{v,sc} \leq Z \tag{10.16}$$

$$\epsilon^{\mathrm{R}}_{v,sc} \leq 5k^{\mathrm{buy}}_v \tag{10.17}$$

$$\text{Minimize } \left(\textstyle\sum_{v\in\mathcal{V}}\sum_{sc\in Sc}\tau_{sc}P_{\mathrm{v,sc}} + \beta_1 P^{\mathrm{R}}_{\mathrm{v,sc}}\right)\beta_2 + Z \tag{11}$$

RESULTS

In this Section, we present the results of our design approach. First, we will present results for the computation of an energy efficient system. Afterwards we will present the results if we also consider the resilience of the system.

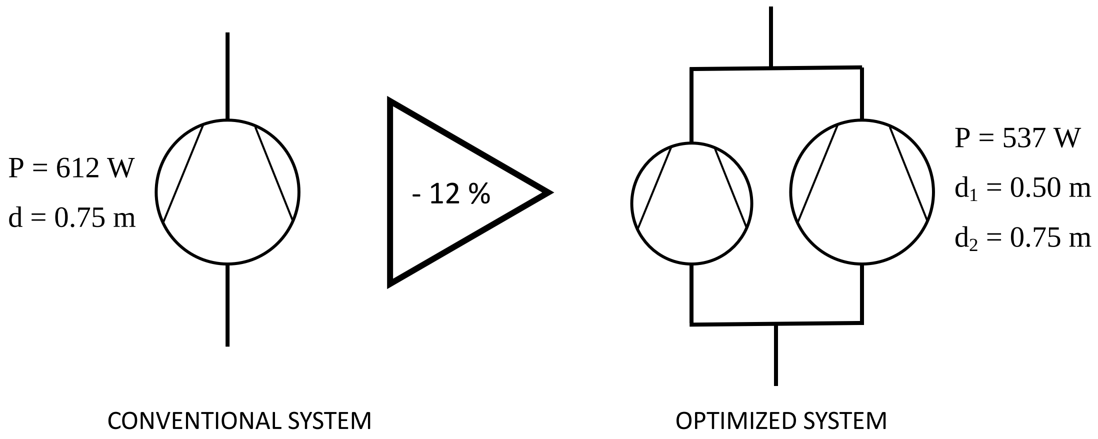

Figure 3: Optimized energy efficient design.

The model for the energy optimization, which is given by Equation (9.1) – (9.14), can be solved by SCIP (cf. [10]) within a few minutes on a standard computer, if we allow a mathematical optimization gap of 5 %. This means, that the solution found is at maximum 5 % away from the global optimal solution.

The result of the energy optimization is shown in Figure 3, detailed values for the single load scenarios are given in Table 5. The optimal system found with TOR consists of two parallel fans with different diameters. The overall power consumption of the optimized system in all loading cases given their respective frequency amounts to 537 W. Compared to 612 W of the conventional system, it is possible to increase the efficiency by round about 12 % by the usage of multiple fans.

Table 4: Results of the efficiency optimization.

Loadcase	Fan number	Fan diameter [m]	Rotational speed [rpm]	Power [W]
1	1	0.50	1347	357
2	2	0.75	816	667
3	2	0.75	924	937

To get a more resilient system, in a second step, we add the additional constraints (10.1) - (10.17) and use the modified objective (11) instead of Equation (9.1). With this extension, the computation of the optimal system is more complex. Therefore, we have to reduce the number of load cases and only consider the first two load scenarios of Table 1. From the optimization, we get multiple solutions that are at least 5 % away from the global optimum and have the same k-reliability. One solution is the same combination as shown before in Table 5: One fan with d = 0.5 m and one fan with d = 0.75 m. In the case of a malfunction of one fan, the other fan replaces this malfunctioning fan in all load scenarios.

Next to this solution, it is also possible to use either fan 1 or fan 2 of Table 4 two times. Again, all scenarios are met by one fan in the case of a failure of the other one. While all solutions have the same k-reliability, the solution of two fans of diameter d = 0.5 m is the most energy efficient, given the first two scenarios of Table 1. During normal usage, it is possible to improve the overall energy efficiency by using both fans. This is shown in Table 5. In the first load scenario, only one fan is used. In scenario two, both fans are used. In comparison, using only one fan in this load case would increase the power consumption by 7.2 %.

Table 5: Results of the optimization of resilience.

Loadcase	Fan Number	Rotational speed [rpm]	Power [W]
1	1	1347	357
2	1 and 2	1245; 1245	311, 311

CONCLUSION

We presented a method to model technical systems and to find optimal topologies regarding energy efficiency and resilience. Therefore, we described a mixed-integer non-linear program to develop a fan system. This model considers, next to the energy efficiency, explicitly the resilience of the underlying ventilation system. The result is a more resilient system with a k-reliability of 1. Further, we showed that especially when considering multiple instead of one single load scenario, the optimal topology found by our method has considerably higher energy efficiency than a standard layout with one single fan.

As next steps, we want to improve the underlying optimization algorithms and mathematical modelling strategies. Our goal is to increase the playing field of the optimization algorithm (i.e. add further fans and dampers), while still maintaining a reasonable solution time, since the usage of more load scenarios and more discrete fans can lead to even higher energy savings.

Additionally we will improve the modelling of resilient systems. While first steps show a promising path to model and design resilient systems, we want to add more load scenarios and more fans in the future.

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ANNEX

Table 6: Model parameters.

Parameter	Value
$\varphi^{\max}$	0.23637
α_1	-28.32336
α_2	-1.70799
α_3	0.20117
α_4	0.0444908
α_5	0.0718617
β_1	0.1
β_2	0.00126994