ON A UNIFORM ROTOR-STATOR WAKE INTERACTION NOISE MODEL BASED ON A MODE-MATCHING TECHNIQUE

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SUMMARY

The proposed paper deals with the analytical modeling of the noise generated by rotor wakes impinging on a row of outlet guide vanes in a ducted subsonic axial-flow fan architecture. The present analytical approach is believed a consistent alternative to numerical methods, especially at the early design stage. The analytical modeling consists in convecting a wake model through the stator and applying a mode-matching technique on its interfaces to determine the generated acoustic waves. This original technique allows introducing simply the cascade effect in a two-dimensional unwrapped representation of the stator in Cartesian coordinates. Its interest is that it can be easily transposed to a three-dimensional annular cascade in cylindrical coordinates.

INTRODUCTION

Axial-flow fan stages, such as the small-size ducted fans used in air-conditioning systems for aircraft or similar technology, are made of a rotor operating with a downstream row of stationary outlet guide vanes (stator). The duct diameter is generally small (around 20 cm) and the rotational speed quite high, leading to a blade-tip Mach number about 0.3. The numbers of rotor blades and stator vanes are quite large, typically between 10 and 30. This makes sound propagation in the duct take place at relatively high frequencies for which numerous higher-order modes are to be taken into account. The wakes issued from the rotor impinge on the vanes, inducing unsteady loads which radiate as equivalent dipoles according to the general background of the acoustic analogy. This mechanism is referred to as wake-interaction noise. It includes tonal noise at multiples of the blade-passing frequency (BPF) associated with the periodic velocity deficit in the wakes and broadband noise associated with wake turbulence. Both contributions need being modeled with a reasonable accuracy and at reasonable computational cost for the sake of low-noise design. Modern computational resources make the tonal noise modeling more and more tractable using numerical methods, such as Unsteady Reynolds-Averaged Navier-Stokes (URANS) simulations. But simulating resolved turbulence is much more difficult, on the one hand, and even the numerical simulation of periodic phenomena remains expensive, on the other hand, especially at the preliminary design stage when numerous configurations must be tested in optimization algorithms. This is why the analytical modeling approach described in the present work is believed an attractive alternative.

Analytical modeling requires drastic approximations on the geometry and on the flow features in order to produce closed-form solutions of interest for engineering purposes. Most often the vanes are assimilated to rigid flat plates of zero thickness embedded in a uniform mean flow. The simplest strategy is to determine the fluctuating lift forces on the vanes with linearized isolated-airfoil theories and then to make the forces radiate. This two-step approach neglects the so-called cascade effect, expectedly dominant for the aforementioned large numbers of vanes. The cascade effect is defined as the effect of adjacent vanes on the aerodynamic and/or acoustic response of one of them. Because the exit flow from the stator must be axial, the vanes are moderately cambered and inclined at the leading edge, and nearly parallel to the axis at the trailing edge. As a result they have a large overlap and quite high solidity (see Fig. (1)-a).

In the present 2D approach, to be considered as a preliminary investigation, the stator is unwrapped and represented by an infinite linear cascade of parallel, axially aligned and zero-stagger plates (Fig. (1)-b). The inter-vane channels are considered as a periodic array of bifurcated waveguides with rigid walls. The same problem could be solved exactly using the Wiener-Hopf technique, as shown by Glegg[1] and readdressed by Posson *et al*[2]. This technique can be extended to a cascade of staggered plates but it cannot be equivalently formulated for an annular cascade. In contrast the present mode-matching technique, partially addressed in a previous work by Ingenito & Roger[3], can be transposed easily in a cylindrical coordinate system to address the configuration of an annular cascade. But it is not suited for an application to staggered vanes. Therefore, till some generalized statement is available, the technique will be attractive as long as the cascade effect is dominant and the stagger effect of secondary importance. This is why it is expected to produce realistic estimates for broadband noise wake-interaction noise, as a result of the averaging inherent to statistical approaches. Yet the formalism allows addressing tonal noise as well.

The following sections detail the analytical formulation of the mode-matching technique applied to the impingement of aerodynamic wakes on the stator vanes. The wake model, the matching equations and sample results are presented. This enables to understand the solving procedure and provides physical insight into the scattering mechanism of the hydrodynamic waves into sound at the leading-edge and trailing-edge interfaces of a stator.

MODE-MATCHING PRINCIPLE

The flow is considered as non-viscous and isentropic. The fluctuations of velocity, pressure and density are solutions of the linearized Euler equations. The mean velocity is axial and uniform, of subsonic Mach number M. All other mean-flow quantities are also constant.

When an oblique incident wave impinges on the front interface of a set of bifurcated channels, the induced azimuthal phase speed along the interface[4] is reproduced in the reflected and transmitted waves to ensure the continuity of the acoustic field. Figure (1) shows a typical annular stator configuration with zero lean and sweep, and a two-dimensional unwrapped representation of the vanes for a cut at radius $r = R_0$. $a = 2\,\pi\,R_0/V$ is the inter-vane distance, V is the number of vanes. An unit oblique incident wave is specified as $\mathrm{e}^{-\mathrm{i}(\omega t - k_x x - k_z z)}$, where k_x is the axial wavenumber and k_z the azimuthal wavenumber. The 2π-periodicity on the variable $\theta = z/R_0$ implies that $2\pi R_0 = n_0 \lambda_z$ or $k_z R_0 = n_0$ where $\lambda_z = 2\pi/k_z$ is the azimuthal wavelength. The reflected waves of the form $\mathrm{e}^{-\mathrm{i}(\omega\, t\, -\, k'_x\, x\, -\, k'_z\, z)}$ must satisfy the same condition: $k'_z\, R_0 = n'_0$. The continuity of the azimuthal phase speed imposes that the phase-shifts between two points in the z direction are the same for all waves. For the incident and the reflected waves, they are respectively given by $e^{(i(2\pi\, n_0)/V)}$ and $e^{(i(2\pi\, n_0')/V)}$, so that $n_0' = n_0 + sV$, $[n_0, s] \in \mathbb{Z}$. The same condition applies to the transmitted duct wave for which a phase-shift between adjacent channels must be equal to $e^{i2\pi\, n_0/V}$.

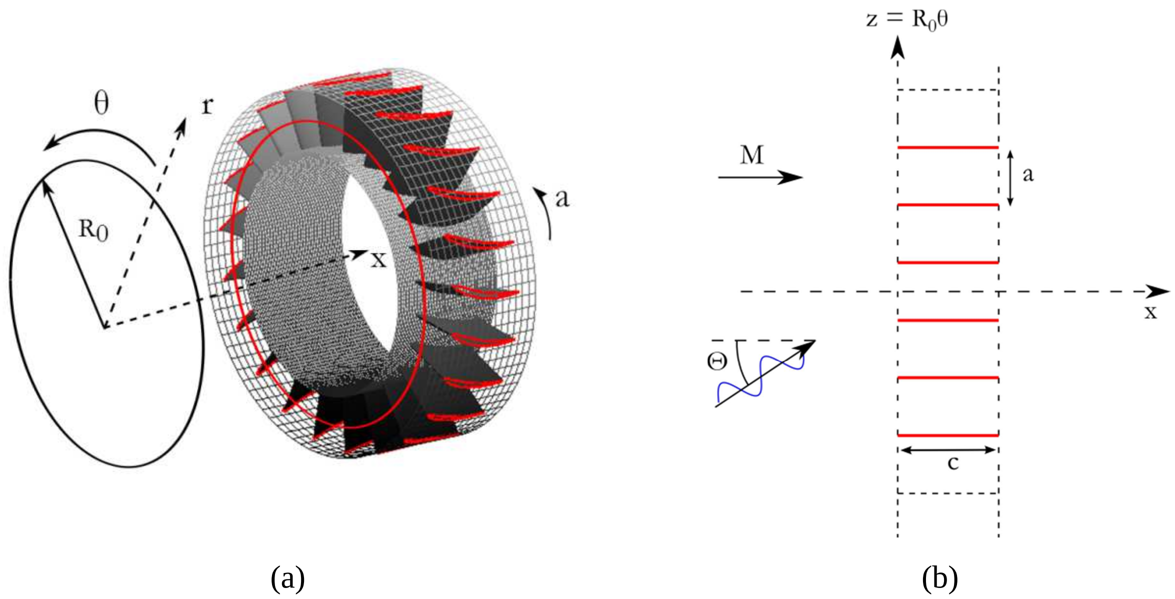


Figure 1 - (a): Typical stator architecture. (b): 2D unwrapped representation of the stator

In the mode-matching technique, the leading-edge and trailing-edge cross-sections of the stator are considered as interfaces at which fluctuating physical quantities are matched to satisfy the basic conservation laws of fluid dynamics. The same principle holds in the original cylindrical coordinates (Fig. (1)-a), as long as all points of a vane edge are approximately contained in the same cross-section, or in a Cartesian unwrapped representation (Figs. ((1)-b, 2)).

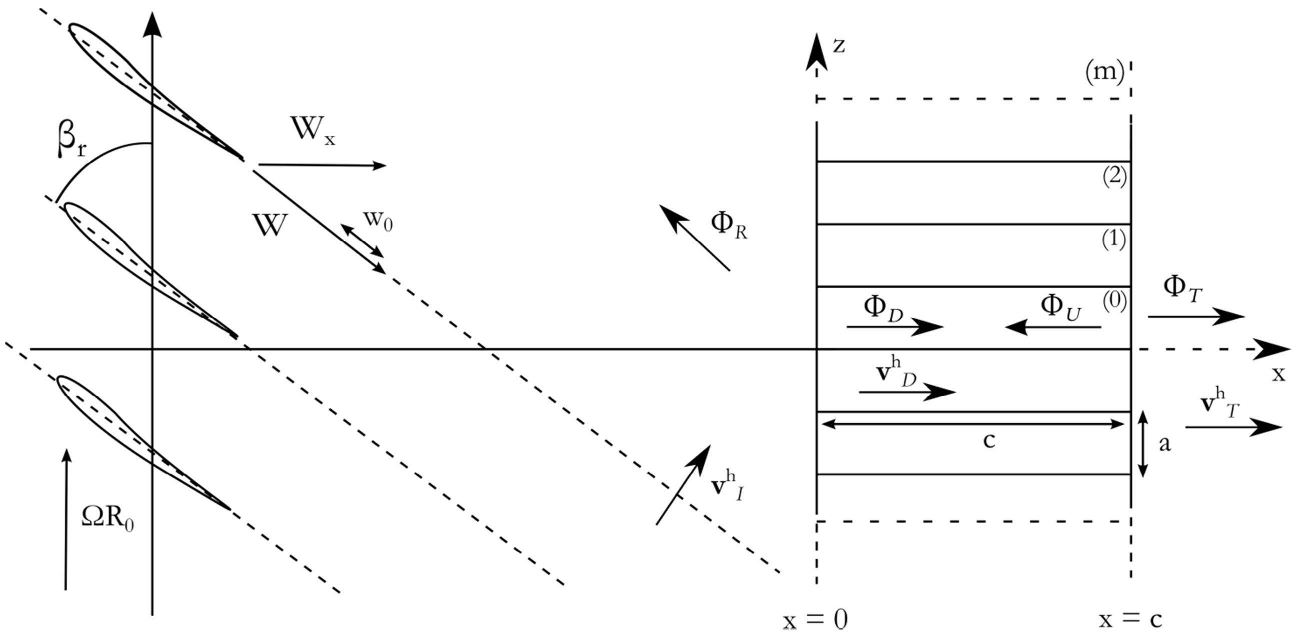


Figure 2 - 2D unwrapped representation of the rotor-stator stage, considered as a bifurcated waveguide system

In general configurations, when the mean-flow quantities have constant but different values on both sides of the interface, jump conditions are expressed on the mass-flow rate and on the total enthalpy (Roger *et al*[5]). In the present study the mean flow is assumed uniform and identical on both sides and the general conditions can be shown to reduce to the continuity of fluctuating pressure and axial velocity. These are classical conditions for the transmission of acoustic waves in axial bifurcated waveguides. But because the linearized equations of gas dynamics for a homogeneous base flow coincide with the convected wave equation they hold also when formulating the acoustic response of an interface to impinging vortical and pressure-free disturbances.

APPLICATION TO WAKE IMPINGEMENT ON A STATOR CASCADE

Wake Modeling

Incident Wakes

The mode-matching technique applied to wake-interaction modeling requires first the definition of the periodic wake velocity deficit induced by the rotor blades on the stator. This incident perturbation is assumed frozen, incompressible and pressure-free, convected by the mean flow. In the absence of more accurate description, the wake velocity deficit is consistently assimilated to a Gaussian function, the parameters of which could be tuned from inspection of the velocity triangle. The Gaussian wake profile was proposed by Reynolds *et al*[6]. The excitation of the stator front face by the periodic wake pattern is reproduced by summing an infinite series of time Gaussian pulses:

$$w(t) = w_0 \sum_{k=-\infty}^{+\infty} \mathrm{e}^{-\xi\left(\frac{t-kT}{\tau}\right)^2} = \sum_{n=-\infty}^{+\infty} w_n \mathrm{e}^{\mathrm{i}nB\Omega t} \tag{1}$$

where $\xi = \ln 2$, w_0 is the velocity deficit on the wake centerline, $\tau = b\,/\,(\Omega R_0)$ is the half-time of the impulse due to a single wake passage, b is the half-width of the wake profile and T is the wake passing period corresponding to the wake passing frequency $B\Omega/(2\pi)$. The spectrum of the wake harmonics decreases with a Gaussian envelope. Furthermore, using the relation $z = (R_0\Omega t)$, and accounting for the convection, the wake velocity deficit becomes:

$$w(x,z) = \sum_{n=-\infty}^{+\infty} w_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z} \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}, \qquad w_n = \frac{w_0 B b}{2\pi R_0}\sqrt{\frac{\pi}{\xi}}\, e^{-\frac{(nBb)^2}{4\xi R_0^2}} .$$

The axial projection of these perturbations is written as:

$$w_x(x,z) = \sin(\beta_r) \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} w_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z} \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}, \quad x \leq 0 \tag{2}$$

The term of order zero must be discarded since it contributes to the non-radiating steady loading on the vanes. Each contribution of order n produces sound at the corresponding multiple of the blade-passing frequency.

Vortical Waves in the Bifurcated Waveguides

This section deals with the transmitted hydrodynamic (vortical) waves in the inter-vane channels. Their definition has to be consistent with the axial convection of the wakes, the rigid-wall condition and the phase-shift between adjacent channels. For any value of n, the axial velocity of the vortical field in the channel of index m is written as a sum of modes:

$$\mathbf{v}_D^h.\mathbf{x} = \sum_{j=0}^{+\infty} A_j^m \cos\left(\frac{j\pi}{a}(z - ma)\right) \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}, \quad 0 \leq x \leq c \tag{3}$$

Adjacent inter-vane channels are phase-shifted by $\mathrm{e}^{\mathrm{i}u}$ with $u = 2\pi nB/V$. If A_j^0 stands for the modal coefficient in the reference channel ($m = 0$), $A_j^m = A_j^0 \mathrm{e}^{\mathrm{i}mu}$ is the coefficient in the channel of index m.

Chu and Kovásznay[7] showed that the acoustic and vortical modes of fluctuation in an isentropic gas remain uncoupled in a linearized theory, provided that the base flow is homogeneous. As a consequence, both develop independently, except at physical boundaries where they couple. In the present case the coupling occurs on the stator vanes, in the sense that the acoustic potential fields combine with the hydrodynamic field *via* the rigid-wall boundary condition. The aforementioned modal expansion automatically fulfills that condition so that the formulation of the coupling is displaced to the interface.

As a result of the Euler equation, the rotational of the hydrodynamic velocity field is conserved through the interface. This is used to determine the amplitudes of the hydrodynamic channel modes. Moreover, the divergence of the hydrodynamic modes is zero, which provides a relationship between the azimuthal and axial velocity components. The velocity field of the incident gust for any value of n and $m = 0$ reads

$$\mathbf{v}_I^h = \begin{cases} \sin(\beta_r) w_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z} \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}\, \mathbf{x} \\ -\dfrac{\Omega R_0}{W_x} \sin(\beta_r) w_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z} \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}\, \mathbf{z} \end{cases}, \qquad x \leq 0 \tag{4}$$

and the velocity field in the inter-vane channel is

$$\mathbf{v}_D^h = \begin{cases} \displaystyle\sum_{j=1}^{+\infty} A_j^0 \cos\left(\frac{j\pi}{a} z\right) \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}\, \mathbf{x} \\ \displaystyle\sum_{j=1}^{+\infty} -\mathrm{i}\frac{nB\Omega a}{j\pi W_x} A_j^0 \sin\left(\frac{j\pi}{a} z\right) \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}\, \mathbf{z} \end{cases}, \qquad 0 \leq x \leq c \tag{5}$$

Matching both expressions of the rotational at the leading-edge interface located at $x = 0$, the following equation is found :

$$\mathrm{i}nB\left(\frac{W_x^2 + R_0^2\Omega^2}{R_0 W_x^2}\right)\sin(\beta_r)\, w_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z} = \sum_{j=1}^{+\infty}\left[\left(\mathrm{i}\frac{nB\Omega}{W_x}\right)^2 \frac{a}{j\pi} - \frac{j\pi}{a}\right] A_j^0 \sin\left(\frac{j\pi}{a} z\right) \mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x} \tag{6}$$

Multiplying both sides by $\sin(\frac{\mu\pi}{a} z)$, $\mu \in \mathbb{Z}$, integrating with respect to z from 0 to a and taking advantage of the orthogonality property of the modes leads to the infinite set of equations

$$\mathrm{i}nB\left(\frac{W_x^2 + R_0^2\Omega^2}{R_0 W_x^2}\right)\sin(\beta_r)\, w_n \Psi_{nB}^{\mu} = \left[\left(\mathrm{i}\frac{nB\Omega}{W_x}\right)^2 \frac{a}{\mu\pi} - \frac{\mu\pi}{a}\right] A_\mu^0 \frac{\pi}{V}, \quad \mu = 1, \dots, N\mu \tag{7}$$

with $\qquad \Psi_{nB}^{\mu} = \dfrac{\mu V\left[(-1)^{\mu}\mathrm{e}^{\mathrm{i}nB\frac{2\pi}{V}} - 1\right]}{2\left((nB)^2 - \left(\frac{\mu V}{2}\right)^2\right)}$ if $nB \neq \frac{\mu V}{2}$, $\qquad \Psi_{nB}^{\mu} = \frac{\mathrm{i}\pi}{V}$ if $nB = \frac{\mu V}{2}$

Thus the modal amplitude of a transmitted hydrodynamic mode is obtained as

$$A_{\mu}^{0} = \frac{\mathrm{i}nB\left(\dfrac{W_x^2 + R_0^2\Omega^2}{R_0 W_x^2}\right)\sin(\beta_r)\, w_n \Psi_{nB}^{\mu}}{\left[\left(\mathrm{i}\dfrac{nB\Omega}{W_x}\right)^2 \dfrac{a}{\mu\pi} - \dfrac{\mu\pi}{a}\right]\dfrac{\pi}{V}} \tag{8}$$

Finally the hydrodynamic wave downstream of the stator is assumed the same as upstream, so that its axial velocity for a given value of n is written as

$$\mathbf{v}_T^h.\mathbf{x} = B_n \mathrm{e}^{\mathrm{i}\left(\frac{nB}{R_0}\right)z}\mathrm{e}^{\mathrm{i}\left(\frac{nB\Omega}{W_x}\right)x}, \quad c \le x \ \text{ where } \ B_n = \sin(\beta_r) w_n \tag{9}$$

A more refined statement paying deeper attention to the way of implementing a Kutta condition at the trailing-edge interface will be addressed in a future work.

Definition of the Acoustic Potentials

The acoustic potentials are solutions of the convected Helmholtz equation:

$$\Delta\phi - \frac{1}{c_0^2}\frac{\mathrm{D}^2\phi}{\mathrm{D}t^2} = \frac{\partial^2\phi}{\partial z^2} + (1-M^2)\frac{\partial^2\phi}{\partial x^2} + 2\mathrm{i}k_n M\frac{\partial\phi}{\partial x} + k_n^2\phi = 0, \quad k_n = \frac{nB\Omega}{c_0}, \tag{10}$$

where the convective derivative is:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \boldsymbol{W}.\boldsymbol{\nabla}$$

Four acoustic fields are produced by the interaction : the “reflected” and the “transmitted” fields (so called because of the upstream/downstream propagation, by identity with the problem of the diffraction of an incident acoustic wave) in the unbounded domains, referred to as ϕ_r and ϕ_t, and those in the inter-vane channels referred to as ϕ_u and ϕ_d for upstream and downstream propagation directions, respectively (Fig. (2)). The reflected (ϕ_r) and the transmitted (ϕ_t) potentials admit a Floquet space-harmonic representation[8]. For the wake harmonic of order n the upstream free-field potential reads

$$\phi_r = \sum_{s=-\infty}^{+\infty} R_s \mathrm{e}^{\mathrm{i}\alpha_{r_s} z}\mathrm{e}^{\mathrm{i}K_{r_s}^{-} x}, \quad x \le 0 \tag{11}$$

with

$$\alpha_{r_s} = \frac{nB + sV}{R_0}, \qquad K_{r_s}^{-} = \frac{-Mk_n - \sqrt{k_n^2 - \beta^2\alpha_{r_s}^2}}{\beta^2}, \qquad M = \frac{W_x}{c_0}, \qquad \beta = \sqrt{1-M^2}$$

The transmitted acoustic potential T is written as

$$\phi_t = \sum_{r=-\infty}^{+\infty} T_r \mathrm{e}^{\mathrm{i}\alpha_{t_r} z}\mathrm{e}^{\mathrm{i}K_{t_r}^{+} x}, \quad c \le x, \quad \alpha_{t_r} = \frac{nB + rV}{R_0}, \qquad K_{t_r}^{+} = \frac{-Mk_n + \sqrt{k_n^2 - \beta^2\alpha_{t_r}^2}}{\beta^2} \tag{12}$$

Both reflection and transmission occur on a set of azimuthal modes which correspond to oblique plane waves in the unwrapped representation. The reflected and transmitted fields result from the modulation of the incident perturbations by the periodicity of the V vanes, according to orders selected by Tyler & Sofrin's rule[9] $nB - sV,\ s \in \mathbb{Z}$.

The upstream and downstream acoustic potentials in the inter-vane channels can be written as

$$\phi_d^m = \sum_{p=0}^{+\infty} D_p^0 \mathrm{e}^{\mathrm{i}mu} \cos\left(\frac{p\pi}{a} z\right) \mathrm{e}^{\mathrm{i}K_{d_p}^+ x}, \qquad 0 \le x \le c, \qquad K_{d_p}^+ = \frac{-Mk_n + \sqrt{k_n^2 - \beta^2 \left(\frac{p\pi}{a}\right)^2}}{\beta^2} \tag{13}$$

$$\phi_u^m = \sum_{q=0}^{+\infty} U_q^0 \mathrm{e}^{\mathrm{i}mu} \cos\left(\frac{q\pi}{a} z\right) \mathrm{e}^{\mathrm{i}K_{u_q}^- x}, \qquad 0 \le x \le c, \qquad K_{u_q}^- = \frac{-Mk_n - \sqrt{k_n^2 - \beta^2 \left(\frac{q\pi}{a}\right)^2}}{\beta^2} \tag{14}$$

where m refers to the m^{th} inter-vane channel. By virtue of the vane-to-vane phase-shift the problem of the determination of the acoustic potentials only needs being solved for the reference channel 0.

Matching Equations

The continuity of the fluctuating pressure and axial velocity is imposed on both interfaces of the stator. Two sets of matching equations can be written at $x = 0$ and $x = c$. The relations between acoustic potential, pressure and velocity are:

$$p^{ac} = -\rho_0 \left(\frac{\partial \phi}{\partial t} + \boldsymbol{W}.\boldsymbol{\nabla}\phi\right), \qquad \mathbf{v}^{\mathrm{ac}} = \boldsymbol{\nabla}\phi \tag{15}$$

For clarity, we consider a vector $\mathbf{\Gamma}$ gathering pressure and axial velocity. Its indices denote the incident (I), reflected (R), transmitted (T), downstream channel (D) and upstream channel (U) hydrodynamic (h) and acoustic (ac) waves.

$$\mathbf{\Gamma}_q(x,z) = \begin{pmatrix} p^{ac}(x,z) \\ v_x^{ac}(x,z) + v_x^h(x,z) \end{pmatrix}, \qquad q = I, R, T, D, U \tag{16}$$

For any value of the wake harmonic n and the channel $m = 0$, the continuity of pressure and axial velocity is imposed at the leading-edge interface ($x = 0$) and the trailing-edge interface ($x = c$). The matching equations read:

$$\mathbf{\Gamma}_I(0,z) + \mathbf{\Gamma}_R(0,z) = \mathbf{\Gamma}_D(0,z) + \mathbf{\Gamma}_U(0,z), \qquad \forall z \tag{17}$$

$$\mathbf{\Gamma}_D(c,z) + \mathbf{\Gamma}_U(c,z) = \mathbf{\Gamma}_T(c,z), \qquad \forall z \tag{18}$$

These equations involve four unkown generic variables ($\mathbf{R}$, $\mathbf{D^0}$, $\mathbf{U^0}$, $\mathbf{T}$) and four matching equations. A global matrix inversion method could be used with a proper truncation but it might not be the best suited because of conditionning issues.

Iterative Method for Multiple Matching

The two sets of equations can be solved by an iterative procedure which has the interest of following the onset of the acoustic response to the hydrodynamic wave. The incident wakes impinging on the leading-edge interface generate upstream (reflected) and downstream (transmitted) acoustic potentials (ϕ_r) and (ϕ_d^0). The latter is partially reflected by the trailing-edge interface, generating the upstream guided field (ϕ_u^0) and the transmitted field (ϕ_t) downstream of the stator. Back-and-forth acoustic waves develop this way in the bifurcated waveguides until convergence. Two steps are required :

- Initialization ($g = 0$)

$$\boldsymbol{\Gamma}_I\,(0,z) + \boldsymbol{\Gamma}_R{}^0(0,z) = \boldsymbol{\Gamma}_D{}^0(0,z), \qquad \forall z \tag{19}$$

$$\boldsymbol{\Gamma}_D{}^0(c,z) + \boldsymbol{\Gamma}_U{}^0(c,z) = \boldsymbol{\Gamma}_T{}^0(c,z), \qquad \forall z \tag{20}$$

- Iterative process ($g > 0$)

$$\boldsymbol{\Gamma}_I\,(0,z) + \boldsymbol{\Gamma}_R{}^g(0,z) = \boldsymbol{\Gamma}_D{}^g(0,z) + \boldsymbol{\Gamma}_U{}^{g-1}(c,z), \qquad \forall z \tag{21}$$

$$\boldsymbol{\Gamma}_D{}^g(c,z) + \boldsymbol{\Gamma}_U{}^g(c,z) = \boldsymbol{\Gamma}_T{}^g(c,z), \qquad \forall z \tag{22}$$

In the initialization step, Eqs. (19) and (20) are solved without upstream wave (vector $\mathbf{U^0} = \mathbf{0}$). Then Eqs. (21) and (22) are solved using the modal coefficients from the downstream channel wave (vector $\mathbf{D^0}$) as input. This provides a first estimate of all modal coefficients. The iterative process is continued until all coefficients $\mathbf{R}$, $\mathbf{D^0}$, $\mathbf{U^0}$ and $\mathbf{T}$ are converged. At every step in this procedure, only two vectors of coefficients ($\mathbf{R}, \mathbf{D^0}$) or ($\mathbf{U^0}, \mathbf{T}$) have to be determined for each interface, which makes the solving easily achievable.

Modal Projection

An usual way of solving the matching equations is to use modal projections. This is illustrated here for the leading-edge interface ($x = 0$). Multiplying each equation by $\cos(\frac{\mu\pi}{a}z)$, $\mu \in \mathbb{Z}$ and integrating with respect to z from 0 to a leads to the infinite sets of equations:

- Continuity of the pressure

$$\sum_{s=-\infty}^{+\infty}\left(k_n - K_{r_s}^- M\right) R_s \Lambda_{\alpha_{r_s}R_0}^{\mu} = \left(k_n - K_{d_\mu}^+ M\right) D_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) + \left(k_n - K_{u_\mu}^- M\right) U_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) \tag{23}$$

- Continuity of the axial velocity

$$\sin(\beta_r)\, w_n \Lambda_{nB}^{\mu} + \sum_{s=-\infty}^{+\infty} \mathrm{i}K_{r_s}^- R_s \Lambda_{\alpha_{r_s}R_0}^{\mu} = \mathrm{i}K_{d_\mu}^+ D_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) + \mathrm{i}K_{u_\mu}^- U_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) + A_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) \tag{24}$$

where

$$\Lambda_{\alpha_\nu}^{\mu} = \frac{\mathrm{i}\alpha_\nu R_0\left[1-(-1)^\mu \mathrm{e}^{\mathrm{i}\frac{2\pi}{V}\alpha_\nu}\right]}{\left[(\alpha_\nu R_0)^2 - \left(\frac{\mu V}{2}\right)^2\right]}, \qquad \alpha_\nu \in \left[\alpha_{r_s}, \frac{nB}{R_0}\right], \qquad \mu = 0, 1, \dots, N_\mu$$

$\delta_{\mu,0}$ stands for the Kronecker symbol. The projection enables to isolate one coefficient in one (or two) infinite sums. Truncating the sum to the order S and recombining the two projected equations in order to eliminate the modal coefficients $\mathbf{D^0}$ leads to a linear system:

$$[\boldsymbol{\Delta}][\mathbf{R}] = [\boldsymbol{\Delta_0}] \tag{25}$$

$$\Delta(\mu,s) = \Lambda_{\alpha_{r_s}R_0}^{\mu}\left[\frac{\left(k_n - K_{r_s}^- M\right)}{\left(k_n - K_{d_\mu}^+ M\right)} - \frac{K_{r_s}^-}{K_{d_\mu}^+}\right]$$

$$\Delta_0(\mu) = U_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right)\left[\frac{\left(k_n - K_{u_\mu}^- M\right)}{\left(k_n - K_{d_\mu}^+ M\right)} - \frac{K_{u_\mu}^-}{K_{d_\mu}^+}\right] + \frac{1}{\mathrm{i}K_{d_\mu}^+}\left[\sin(\beta_r)\, w_n \Lambda_{nB}^{\mu} - A_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right)\right]$$

$$\mu = 0, 1, \dots, N_\mu; \quad s = -S, \dots, 0, 1, \dots, S$$

R is the vector of the modal coefficients of the “reflected” waves, of size $2S+1$. This equation is then solved by matrix inversion. Then, the modal coefficients for the downstream channel waves $\mathbf{D^0}$ are expressed from **R** :

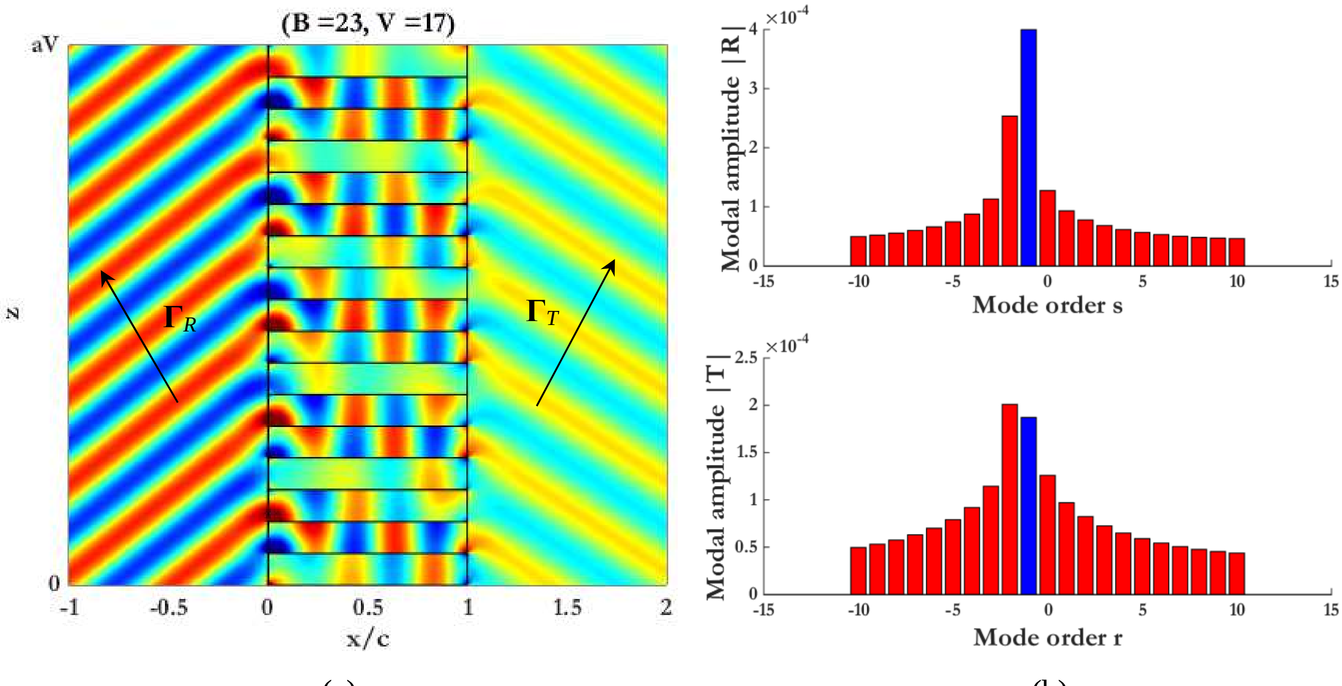
$$\left[D_\mu^0\right] = \frac{V}{\mathrm{i}K_{d_\mu}^+ \frac{\pi}{V}\left(1+\delta_{\mu,0}\right)}\left[\sum_{s=-\infty}^{+\infty} \mathrm{i}K_{r_s}^- R_s \Lambda_{\alpha_{r_s} R_0}^{\mu} - \mathrm{i}K_{u_\mu}^- U_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right) + \sin(\beta_r)\; w_n \Lambda_{nB}^{\mu}\left(1+\delta_{\mu,0}\right) \;\; - A_\mu^0 \frac{\pi}{V}\left(1+\delta_{\mu,0}\right)\right] \tag{26}$$

The same procedure is applied for the trailing-edge interface.

RESULTS

Rotor-Stator Interaction

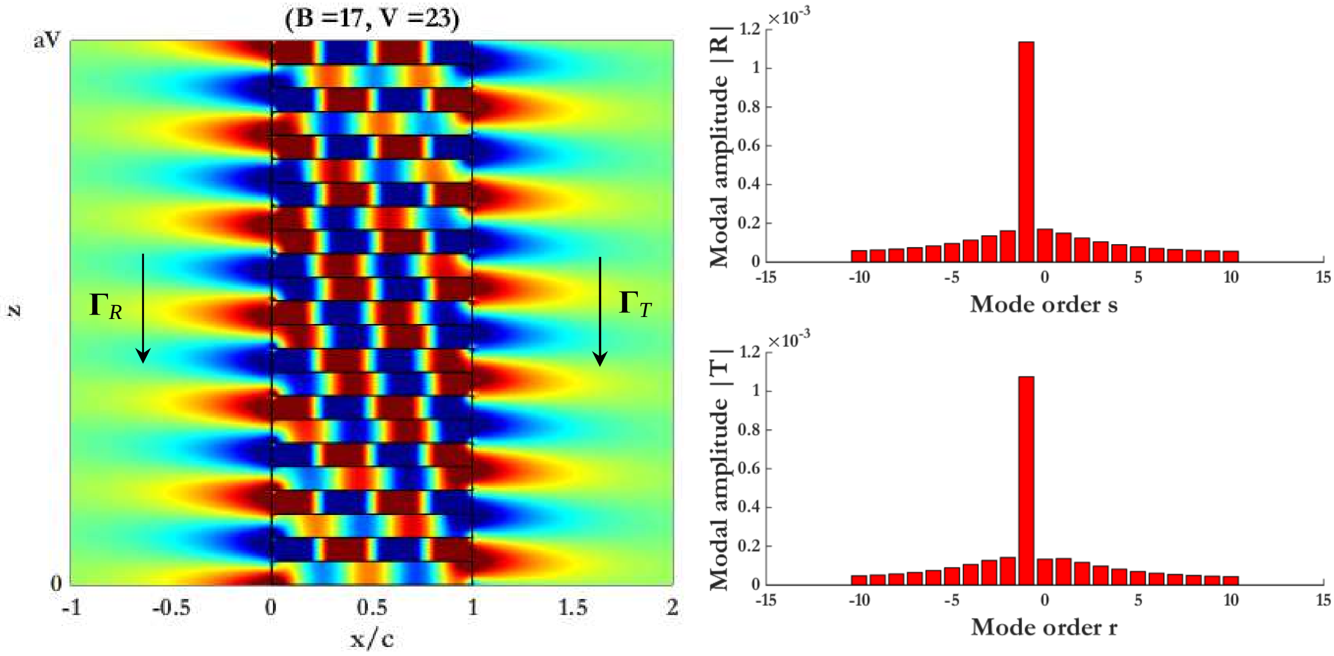
Typical results are reported in Figs. (3) and (4). Only the first Fourier component of the incident wake pattern is considered. It corresponds to the Blade Passing Frequency (BPF) of the rotor.



(a) (b)

Figure 3 - (a): Typical pressure field generated by wake impingement on the front face of a finite-chord stator cascade, at the BPF. Upstream and downstream emissions of an oblique wave (cut-on). Full unwrapped annulus. (b): Modal amplitudes |R| and |T|. Blue (red) bars denote cut-on (cut-off) modes.

The plotted quantity is the instantaneous pressure, shown for qualitative illustration. The incident wakes travelling upwards have no trace in terms of pressure. The parameters of the system are $B = 23, V = 17,\ \Omega\ =\ 11000\ \text{rpm},\ W_x\ =\ 20\ \text{m/s},\ \beta_r = \pi/6$ and $c\ =\ 0.2\ \text{m}$. The non-realistic value of the chord is chosen to illustrate the acoustic propagation in the channels. The rotor-stator system ($B = 23, V = 17$, Fig. (3)) emits upstream and downstream propagating oblique waves in the annular duct. These waves correspond to the dominant co-rotating mode $n_i = 23 - 17 = +6$ of phase speed $23\,\Omega\,R_0/6$ expected from Tyler & Sofrin's rule. A plane-wave mode is transmitted and reflected in each inter-vane channel, with a phase shift between adjacent channels that reproduces the same $n_i = 6$ periodicity as for the upstream mode.



(a) (b)

Figure 4 - (a): Typical pressure field generated by wake impingement on the front face of a finite-chord stator cascade, at the BPF. Condition of total cut-off upstream and downstream emissions. Full unwrapped annulus. (b): Modal amplitudes |R| and |T|. Red bars denote cut-off modes.

For the rotor-stator system ($B = 17, V = 23$, Fig. (4)) at the first BPF, the interaction generates the dominant mode $n_i = 17 - 23 = -6$. This mode is contra-rotating, which means that the phase speed $-17\Omega\,R_0/6$ is downwards on the plot. In this case there is no oblique upstream and downstream propagation. The amplitude of the mode is exponentially attenuated from the interfaces, with axially aligned pseudo-wavefronts. This corresponds to a cut-off mode. In contrast the plane-wave mode is again transmitted and reflected back and forth in the inter-vane channels with a phase-shift between adjacent channels. The tangential phase speed along the interface is supersonic in the first case (Fig. (3)) and subsonic in the second case (Fig. (4)), with respect to the fluid.

Parametric Study: Vane Chord Variation

The effect of the vane chord variation on the radiated acoustic power is illustrated in Fig. (5). The chord ($c = 0.2$ m) is increased by one centimeter. The rotor-stator configuration ($B = 23, V = 17$) which emits propagating oblique waves is considered. All parameters remain the same. The acoustic powers P_j, ($j = R, T$) per unit span of the upstream and downstream waves of potentials ϕ_r, ϕ_t respectively, are defined as[10]

$$P_R = \frac{Z_0 aVk_n}{2} \sum_{\substack{s=-\infty \\ cut-on\ modes}}^{+\infty} \overline{k_{r_s}}\,|R_s|^2, \ \ \overline{k_{r_s}} = \sqrt{k_n^2 - \beta^2\alpha_{r_s}^2} \tag{27}$$

$$P_T = \frac{Z_0 aVk_n}{2} \sum_{\substack{r=-\infty \\ cut-on\ modes}}^{+\infty} \overline{k_{t_r}}\,|T_r|^2, \ \ \overline{k_{t_r}} = \sqrt{k_n^2 - \beta^2\alpha_{t_r}^2} \tag{28}$$

where $Z_0 = \rho_0 c_0$ is the acoustic impedance of the fluid. $\overline{k_{r_s}}$, $\overline{k_{t_r}}$ are respectively the propagative parts of the axial wavenumbers of the upstream and downstream waves. Only the cut-on modes carry energy and contribute in the expression of the acoustic power.

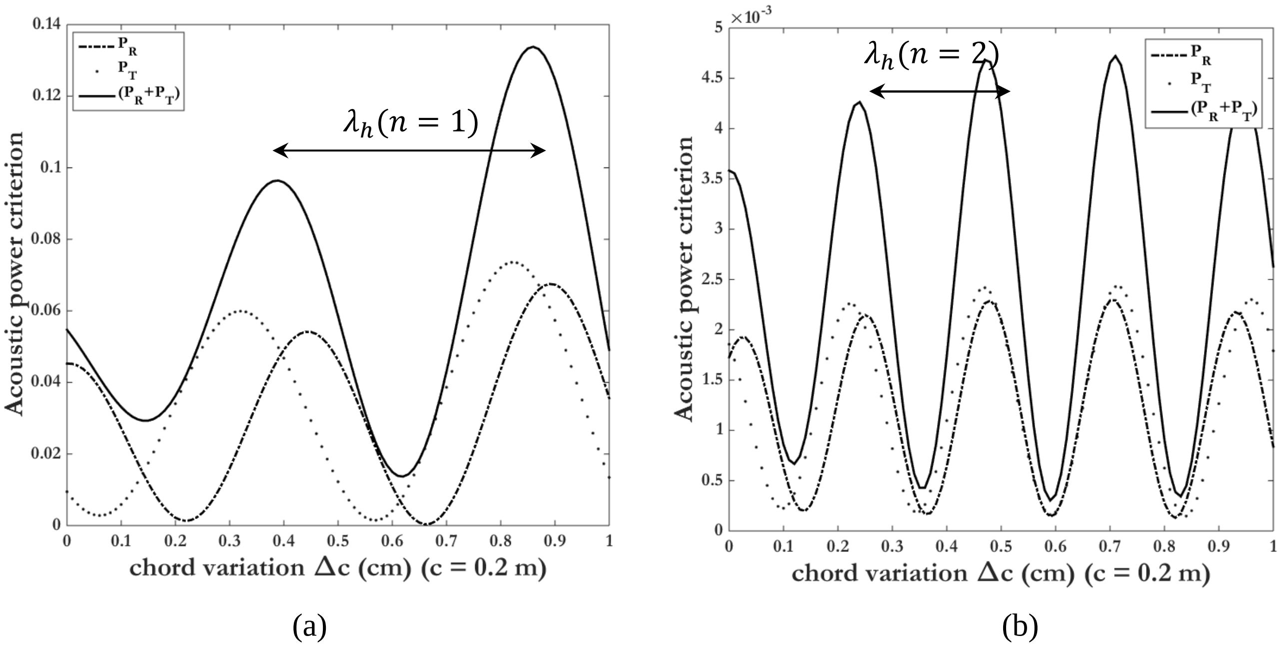


Figure 5 – Effect of a chord variation on the upstream (dashed), downstream (dotted) and total (solid) radiated acoustic powers per unit span. (a): at the BPF (n=1), (b): at 2 BPF (n=2)

Figure (5) shows a nearly periodic variation of the acoustic power with increasing chord length. Between Fig. (5)-a (BPF) and Fig. (5)-b (2 BPF), the period of the variation is divided by two. These periods are approximately equal to the axial hydrodynamic wavelength:

$$\lambda_h = \frac{2\pi W_x}{nB\Omega} \tag{29}$$

At the BPF and 2 BPF, the axial hydrodynamic wavelengths are $\lambda_h(n=1) = 4.7$ mm and $\lambda_h(n=2) = 2.4$ mm. The variation denotes a resonance effect driven by the relative phases of the hydrodynamic waves at both interfaces. Figure (5) shows that the acoustic radiation generated by the impact of a hydrodynamic wave on a stator is dependent on the ratio between the chord length and the axial wavelength of the wake perturbations.

CONCLUSION AND FUTURE WORK

An original mode-matching technique dedicated to the modeling of wake-interaction noise production in an axial-flow fan stator vane row has been described in the paper. The technique allows simply accounting for the cascade effect of adjacent vanes. In addition to classical matching equations of acoustics involving the continuity of axial velocity and pressure, a specific conservation equation for the vorticity is considered. The preliminary implementation and first results in a two-dimensional unwrapped reduction of the stator confirm that it is relevant for an efficient prediction of the sound. The sound waves are directly expressed in a modal form from a modal expansion of the wake disturbances, without intermediate step. In particular the model provides a direct way of illustrating the cut-on and cut-off waves produced by wake interactions according to Tyler & Sofrin's rule. The other interest is that the approach could be declined in a three-dimensional annular cascade in cylindrical coordinates, which will be addressed in a future work. It will also be declined in a statistical approach to formulate the broadband noise associated with wake interactions.

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