SIMPLIFIED THEORY OF A RISK OF THERMAL CATASTROPHE IN AIR-COOLED ELECTRONIC CIRCUITS

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SUMMARY

The paper illustrates the importance of adequate air cooling systems in computers and other electronic systems with negative temperature coefficients of resistance (typical for the elements made of carbon and semiconductors). For those electric circuits a simplified mathematical model has been applied to demonstrate a high risk of thermal catastrophe: when no thermal equilibrium exists and the circuit temperature grows up infinitely. Certainly, the real-life effects are more complex, but the theory evidently describes the main trends.

INTRODUCTION

One of the important contemporary applications of fans is to cool computers and temperature-sensitive electronic devices. As known, the heat produced by electrical current can notably elevate the circuit temperature and damage its components (for instance, Central Processing Units may generate a notable heat and crash if overheated). The risk is high for electric circuits with a negative temperature coefficient of resistance [1], in particular, in electronics made of semiconductors (silicon and germanium). The goal is to interpret this important effect in simple terms, understandable to students and practical engineers.

SIMPLIFIED MATHEMATICAL MODEL

Consider a simplified electrical circuit (Fig. 1) consisting of a constant voltage source and resistor with the electrical resistance

$$R = R_0 (1 + \alpha t) \qquad (1)$$

where R_0 is the resistance at Celsius scale temperature t, α is the temperature coefficient of resistance which is positive for most of conductors (that is, the electrical resistance increases with temperature). However, for graphite, amorphous carbon, and semiconductors (in particular, silicon and germanium), the temperature coefficient of resistance is negative because their electrical resistance decreases with temperature. It is noteworthy that the first filaments for electric bulbs were made of carbon or carbonized material [2]. Amorphous carbon, a form of graphite consisting of microscopic crystals, is obtained by heating any of a variety of carbon-rich materials (wood, coal, *et cetera*) in a limited amount of air so that complete combustion would not occur.

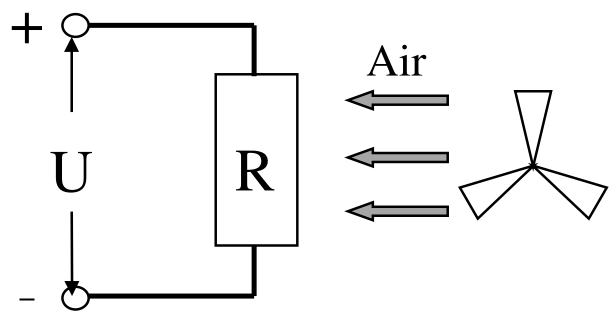

Figure 1: simplified model of air- cooled electrical circuit

Generally, Eq. (1) is a simplified approximation, which is feasible only if $\alpha\, t > -1$, that is,

$$\begin{cases} t > -\dfrac{1}{\alpha}, & \alpha > 0, \\ t < \dfrac{1}{|\alpha|}, & \alpha < 0. \end{cases} \qquad (2)$$

Let the thermal power transferred from the resistor into ambient air be given by equation

$$P_{out} = A\, t \qquad (3)$$

where A is the surface heat conductance, or coefficient of heat transfer [1]. Eq. (2) is valid under the following conditions: (1) thermal convection is a dominant mechanism of heat transfer, (2) the Celsius scale temperature of ambient air equals zero. The second condition is not important and is used just to simplify the related mathematics. In particular, the study is valid for a non-zero ambient temperature if: (1) the value t represents the difference between the circuit temperature and ambient temperature t_{amb}, (2) R_0 is the circuit resistance at the ambient temperature t_{amb}; (3) the coefficient α is replaced with the value $\dfrac{\alpha}{1+\alpha\, t_{amb}}$.

The heat input is given by

$$P_{in} = \frac{U^2}{R} \qquad (4)$$

where U is the constant voltage applied to the resistor. Using Eqs (1), (3), and (4), the condition of thermal equilibrium $(P_{in} = P_{out})$ is written in the form

$$\frac{U^2}{R_0(1+\alpha\, t)} = A\, t \qquad (5)$$

that can be transformed into quadratic equation

$$\alpha t^2 + t - \frac{U^2}{R_0 A} = 0$$

that has two solutions (roots)

$$t_{1,2} = \frac{-1 \pm \sqrt{1+D}}{2\alpha} \tag{6}$$

where the dimensionless parameter

$$D = \frac{4\alpha U^2}{R_0 A}. \tag{7}$$

Consider two main cases:

(1) the temperature coefficient of resistance is positive $(\alpha > 0)$,
(2) the temperature coefficient of resistance is negative $(\alpha < 0)$.

THERMAL EQUILIBRIUM FOR $\alpha > 0$

In this case $D > 0$, so, just the positive one of two solutions (6) is feasible:

$$t_1 = \frac{-1 + \sqrt{1+D}}{2\alpha}. \tag{8}$$

The other formal solution $t_2 = -\frac{1 + \sqrt{1+D}}{2\alpha} < -\frac{1}{\alpha}$ is inconsistent with the conditions (2). In the asymptotic case $\alpha \to 0$, $t_1 \to \frac{U^2}{R_0 A}$, so the temperature increases with the voltage squared. But if the dimensionless parameter $D >> 1$ (as may happen for the high voltage and/or low surface heat conductance), $t_1 \to \frac{U}{\sqrt{R_0 A \alpha}}$ so the temperature grows just linearly with the voltage. Based on the mentioned above, the electrical circuits with a positive temperature coefficient of resistance are to some extent inertial thermally. The thermal equilibrium at the temperature given by Eq. (8) is stable but can be excessive to cause damage. Therefore, the cooling is needed if the voltage is too high.

THERMAL EQUILIBRIUM FOR $\alpha < 0$

Now, Eq. (6) can be expressed in the form

$$t_{1,2} = \frac{1 \pm \sqrt{1 - |D|}}{2|\alpha|} \tag{9}$$

with three important cases: $|D| < 1$, $|D| = 1$, and $|D| > 1$.

In case $|D| < 1$, Eq. (9) has two real roots indicating two thermal equilibriums but, as will be proved later, only one of them, at temperature $t_1 = \frac{1 - \sqrt{1 - |D|}}{2|\alpha|}$, is stable; the equilibrium at temperature $t_2 = \dfrac{1+\sqrt{1-|D|}}{2|\alpha|}$, which is still consistent with the conditions (2), is unstable. The stability can be checked graphically by plotting the left and right parts of Eq. (5) in the 2-D coordinate system for temperature (X) and thermal power (Y). The left part, which describes the heating power, is a hyperbola. The right part is a straight line through the coordinate origin; three such straight lines with different slopes are plotted in Fig. 2: the larger the slope, the higher the surface heat conductance and cooling effectiveness.

The High Cooling line (with the highest slope) intersect the hyperbola at two points; this is a case of two roots t_1 and t_2 attained if $|D|<1$. Here, if the temperature drops below t_1 (for instance, down to temperature $tA < t_1$, the heat input represented by the hyperbola exceeds the heat output represented by the High Cooling straight line); as a result, the temperature of the resistor increases to the equilibrium point t_1. If the temperature of the resistor goes up to temperature $tB > t_1$, the heat input exceeds the heat output and the temperature of the resistor reduces to the equilibrium point t_1. Hence, this equilibrium is stable.

If the temperature of the resistor goes up to temperature $tC > t_2$, the heat input curve is above the High Cooling straight line, and the temperature should go up infinitely. Therefore, the equilibrium at temperature t_2 is unstable.

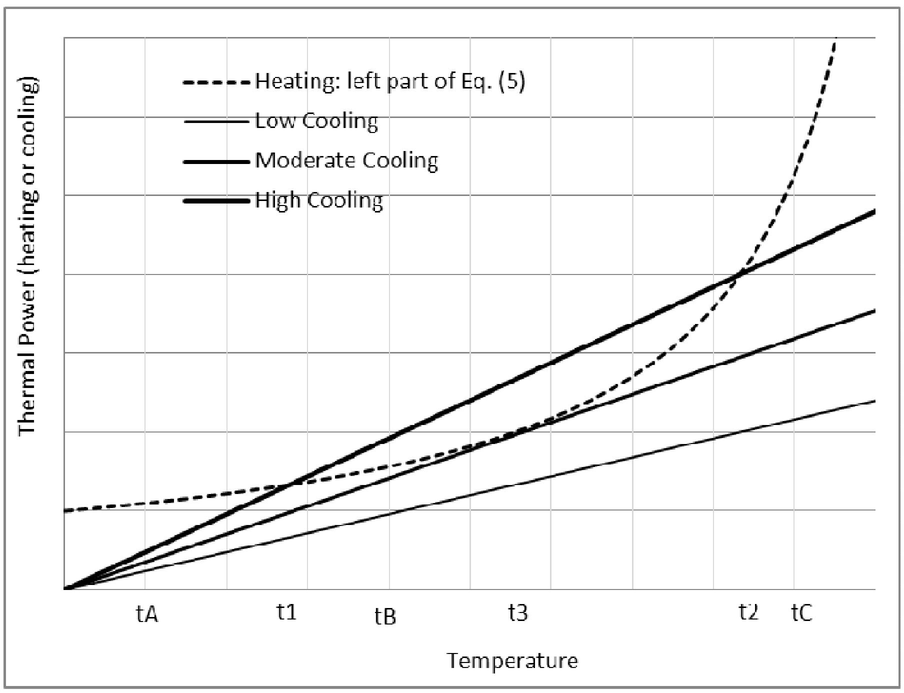

Figure 2: heating and cooling in an electrical circuit with negative temperature coefficient of resistance

In case $|D|=1$, Eq. (9) has just one root $t_3 = \dfrac{1}{2|\alpha|}$, and this equilibrium is unstable.

In case $|D|>1$, Eq. (9) has no real roots, so, no thermal equilibrium exists, and the temperature rises up infinitely. Such an unfavorable transient process is not described in the monograph [3] but can be considered like a thermal catastrophe.

CONCLUSION

In contrast to the electric circuits with a positive temperature coefficient of resistance, the circuits with a negative temperature coefficient of resistance (common for electronics) are at a higher risk of thermal catastrophe. Using a simple mathematical model, the following theoretical results were obtained: (1) if the temperature coefficient of resistance is negative but the air cooling is sufficient, the thermal equilibrium may formally exist at two temperatures, but only one of such equilibriums, which is at a lower temperature, is stable; (2) if the temperature coefficient of resistance is negative and the air cooling is insufficient, a thermal equilibrium is not feasible at all, and the temperature goes up infinitely; (3) for comparison, if the temperature coefficient of resistance is positive, a stable equilibrium is always achieved but the temperature can be high or low depending on the cooling rate. Certainly, the real-life conditions are more complex but this simple theory evidently describes the main trends and may help practical specialists and students understand the particular importance of adequate air cooling systems in computers and other electronic devices.

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